

Low-Complexity Iterative Detection for Uplink Multiuser Large-Scale MIMO

Mojtaba Amiri

Department of Electrical Engineering, Shahed University, Tehran, Iran
mojtaba.amr72@gmail.com

Mahmoud Ferdosizade Naeiny*

Department of Electrical Engineering, Shahed University, Tehran, Iran
m.ferdosizade@shahed.ac.ir

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Abstract

In massive Multiple Input Multiple Output (MIMO) or large scale MIMO systems, uplink detection at the Base Station (BS) is a challenging problem due to significant increase of the dimensions in comparison to ordinary MIMO systems. In this letter, a novel iterative method is proposed for detection of the transmitted symbols in uplink multiuser massive MIMO systems. Linear detection algorithms such as minimum-mean-square-error (MMSE) and zero-forcing (ZF), are able to achieve the performance of the near optimal detector, when the number of base station (BS) antennas is enough high. But the complexity of linear detectors in Massive MIMO systems is high due to the necessity of the calculation of the inverse of a large dimension matrix. In this paper, we address the problem of reducing the complexity of the MMSE detector for massive MIMO systems. The proposed method is based on Gram Schmidt algorithm, which improves the convergence speed and also provides better error rate than the alternative methods. It will be shown that the complexity order is reduced from $O(n_t^3)$ to $O(n_t^2)$, where n_t is the number of users. The proposed method avoids the direct computation of matrix inversion. Simulation results show that the proposed method improves the convergence speed and also it achieves the performance of MMSE detector with considerable lower computational complexity.

Keywords: Massive MIMO; Iterative Method; Matrix Inversion; Maximum Likelihood; MMSE Detection.

1- Introduction

In the recent years massive multiuser multiple-input and multiple-output (MIMO) or large scale MIMO technology has been suggested for next generation wireless communication systems. In massive MIMO systems, a large number of antennas are used at the base station (BS). This structure makes it possible to detect the transmitted symbols of several users that they are transmitting their symbols at the same time and frequency. Massive MIMO is one of the promising solutions in future wireless communication systems (such as 5G systems) that increases spectral efficiency and it reduces interference [1-6]. In spite of the benefits of massive MIMO systems, there exist several challenges in these systems such as hardware implementation complexity, detection complexity, channel estimation and antenna correlation [7, 8].

The maximum likelihood (ML) detector is the optimal detector which practically is not feasible for a massive MIMO system, due to the exponential increase in computational complexity with increasing number of

antennas [9, 10]. Therefore, suboptimal detectors with lower complexities are suggested. To achieve near-optimal performance, the sphere decoding and K-best methods have been suggested for ordinary MIMO systems. But, these methods are not practical for Massive MIMO systems [8, 11, 12].

Linear detection algorithms such as zero-forcing (ZF) and minimum-mean-square-error (MMSE) receivers can achieve a close performance to that of optimal detector in massive MIMO systems, due to the asymptotic orthogonal channel property [2, 13, 14]. Therefore, linear detection algorithms can be employed in massive MIMO systems with a large number of antennas at the BS. Unfortunately, these methods are involved the inversion of a matrix with the size of the number users, which it imposes a considerable computational cost at the receiver side, due to the large number of users [15].

Recently, different algorithms have been proposed to avoid calculating high-dimensional matrix inversion, for example: Gauss-Seidel (GS) [16], Joint Steepest Descent and Jacobi method (JSDJD) [17], Parallelizable Chebyshev Iteration (PCI) [18], Hybrid Iteration Method (HIM) [19], Neumann series (NS) [20-22]. But

* Corresponding Author

performance of these methods is far away from the optimal detector performance [23, 24].

In this paper, a low complexity method has been proposed based on Gram Schmidt method. The proposed method avoids the direct computation of matrix inversion in MMSE detector. The numerical results verify that the proposed detector achieves the near-optimal performance of the MMSE detector with a significantly reduced computational complexity. Performance of the proposed method has been compared with GS, JSDJD, PCI and HIM methods.

The rest of the paper is organized as follows. Section 2 describes the system model of uplink multiuser massive MIMO system. In section 3 the proposed detector and convergence analysis are presented. Complexity analysis is shown in Section 4. In Section 5, the simulation results and discussions about the performance of the proposed algorithm are presented and finally the paper is concluded in Section 6.

Notation: Boldface capital letters and lowercase letters represent matrices and vectors, respectively. \mathbf{I}_K denotes the $K \times K$ identity matrix; $(\cdot)^H$, $(\cdot)^{-1}$, $(\cdot)^T$ and $\|\cdot\|$ denotes conjugate transposition, inversion, transposition and the Euclidean norm of a vector, respectively. $\mathbb{C}^{i \times j}$ denote the set of all $i \times j$ complex matrices.

2- System Model

We consider uplink multiuser massive MIMO system with n_t single antenna users transmitting data to a base station with n_r antennas. The transmitted vector $\mathbf{x} = [x_1, x_2, \dots, x_{n_t}]^T$ includes n_t data symbols that the elements of \mathbf{x} come from the M-QAM constellation with average power σ_x^2 per symbol i.e. $E\{\mathbf{x}\mathbf{x}^H\} = \sigma_x^2 \mathbf{I}_{n_t}$. The received vector at the BS can be represented by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1)$$

Where $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ is the channel matrix between the BS and the n_t users, whose entries are modeled as independent and identically distributed (iid) complex Gaussian random variables with zero mean and unit variance, and \mathbf{n} is a white Gaussian noise vector with zero mean and correlation matrix of $E\{\mathbf{n}\mathbf{n}^H\} = \sigma^2 \mathbf{I}_{n_r}$, where σ^2 is the variance of the noise. It is assumed that channel matrix is known perfectly at the BS, but it is unknown at the transmitter.

The BS detects the transmitted symbols, \mathbf{x} , knowing the received vector, \mathbf{y} and the channel matrix, \mathbf{H} . Since in massive MIMO systems the number of users, n_t , and the number of BS antennas, n_r , may be in order of hundreds, detection methods which are conventionally used in typical (low scale) MIMO systems are not applicable in

massive MIMO scenario.

2-1- MMSE Detection

The MMSE detector is a linear detector which minimizes the mean square error between the transmitted vector and its estimation. The MMSE estimation of the vector \mathbf{x} can be denoted by [16]

$$\hat{\mathbf{x}} = \left(\mathbf{H}^H \mathbf{H} + \frac{\sigma^2}{\sigma_x^2} \mathbf{I}_{n_t} \right)^{-1} \mathbf{H}^H \mathbf{y} = \mathbf{A}^{-1} \mathbf{y}_{MF} \quad (2)$$

Where $\mathbf{y}_{MF} = \mathbf{H}^H \mathbf{y}$ is the output of matched filter and $\mathbf{A} = \mathbf{H}^H \mathbf{H} + \frac{\sigma^2}{\sigma_x^2} \mathbf{I}_{n_t}$ is the MMSE filtering matrix. It is noteworthy that, the MMSE detection generally is not optimum, but it has been shown that its performance in massive MIMO systems, with a large number of antennas, is very close to optimum Maximum Likelihood (ML) detector [17].

Since in massive MIMO systems, the dimension of matrix $\mathbf{A}_{n_t \times n_t}$ is very large, the inversion of this matrix in (2) is very complex. To avoid calculating the inversion of matrix \mathbf{A} , which has the complexity of $O(n_t^3)$, the solution of the following linear equation is found using iterative methods [25].

$$\mathbf{A}\mathbf{x} = \mathbf{y}_{MF} \quad (3)$$

When $n_r \rightarrow \infty$, \mathbf{A} becomes diagonal dominant, which means $|a_{ii}| > \sum_{j \neq i} |a_{ij}| \forall i$, where a_{ij} is the element of the i th row and the j th column of the matrix when the matrix \mathbf{A} is diagonal dominant, some iterative methods can be used to solve (3) instead of using the direct matrix inversion.

3- Proposed Detection Algorithm

In this section, an iterative detector based on the Gram Schmidt method is proposed for detection of the transmitted symbols in massive MIMO systems. This method iteratively achieves the near-optimal performance of the MMSE detector without calculation the matrix inversion.

The main advantage of this method is that it converges faster than the previously proposed methods such as NS [20], HIM [19], GS [16] and JSDJD [17]. In [26, 27] the well-known conjugate Gram Schmidt method has been used to solve an approximated solution of linear equations. We have used this method to solve (3) and to achieve estimation of transmitted symbols in massive MIMO systems. The pseudo-code of the proposed algorithm has been shown in Algorithm 1.

| |
|---------------------------------------|
| Algorithm 1: Proposed Detector |
|---------------------------------------|

Input: \mathbf{y} , \mathbf{H} , σ^2 and σ_x^2
 parameters: K ‘number of iterations’
Output: The final estimation of transmit symbols: $\hat{\mathbf{x}}_{K+1}$

initialization:

1: $\mathbf{A} = \mathbf{H}^H \mathbf{H} + \frac{\sigma_x^2}{\sigma^2} \mathbf{I}_{n_t}$ and $\mathbf{D} = \text{diag}(\mathbf{A})$

2: $\hat{\mathbf{x}}_1 = \mathbf{D}^{-1} \mathbf{y}$ ‘Initial Estimation’

3: $\mathbf{r}_1 = \mathbf{y} - \mathbf{H} \hat{\mathbf{x}}_1$ and $\mathbf{p}_1 = \mathbf{r}_1$

Iteration:

4: $\mathbf{d}_i = \mathbf{A} \mathbf{p}_i$

5: $a_i = \frac{\mathbf{p}_i^H \mathbf{r}_i}{\mathbf{p}_i^H \mathbf{d}_i}$

6: $\hat{\mathbf{x}}_{i+1} = \hat{\mathbf{x}}_i + a_i \mathbf{p}_i$

7: $\mathbf{r}_{i+1} = \mathbf{r}_i - a_i \mathbf{d}_i$

8: $\mathbf{p}_{i+1} = \mathbf{r}_{i+1} - \sum_{j=1}^i \frac{\mathbf{r}_{i+1}^H \mathbf{d}_j}{\mathbf{p}_j^H \mathbf{d}_j} \mathbf{p}_j$

9: if $i > K + 1$ break else go back to step 4

10: **return** $\hat{\mathbf{x}}_{K+1}$

The proposed algorithm is initialized using steps 1, 2 and 3. In these steps, the MMSE filtering matrix \mathbf{A} , is calculated. Then in step 2 an estimation of vector \mathbf{x} is calculated using the inversion of \mathbf{D} instead of inversion of \mathbf{A} :

$$\hat{\mathbf{x}}_1 = \mathbf{D}^{-1} \mathbf{y} \quad (4)$$

\mathbf{D} is defined as a diagonal matrix which its diagonal elements is equal to the diagonal elements of the matrix \mathbf{A} . Sine \mathbf{D} is diagonal the calculation of its inversion in (4) is not complex.

As mentioned before in massive MIMO systems the matrix \mathbf{A} is diagonal dominant, thus $\hat{\mathbf{x}}_1$ is a good estimation for initialization of the algorithm.

In the $(i + 1)$ th iteration, the estimated vector $\hat{\mathbf{x}}_{i+1}$ is calculated by adding a vector in direction of \mathbf{p}_i to the previous estimation $\hat{\mathbf{x}}_i$.

$$\hat{\mathbf{x}}_{i+1} = \hat{\mathbf{x}}_i + a_i \mathbf{p}_i \quad (5)$$

where $\hat{\mathbf{x}}_i$ is the estimated vector after the i th iteration and \mathbf{p}_i is search direction vector. The search directions \mathbf{p}_i is chosen such that the residual error in the $(i + 1)$ th iteration should be orthogonal to \mathbf{p}_i ($\mathbf{p}_i^H \mathbf{e}_{i+1} = 0$) [27]. Since the error vector is unknown, in [27], the Gram Schmidt orthogonalization process is used to find the \mathbf{A} -orthogonal search direction vector \mathbf{p}_i , where \mathbf{A} -orthogonal is defined as follows:

$$\mathbf{p}_i^H \mathbf{A} \mathbf{p}_j = 0, \quad \forall j < i \quad (6)$$

The interpretation of (6) is that each direction, after the multiplication by the matrix \mathbf{A} is orthogonal to the direction vectors of the previous iterations.

In [27] it has been shown that the search direction vector \mathbf{p}_i is calculated by

$$\mathbf{p}_i = \mathbf{r}_i - \sum_{j=1}^{i-1} \beta_{ij} \mathbf{p}_j \quad (7)$$

which means that \mathbf{p}_i is generated by the subtraction of the previous directions \mathbf{p}_j , ($j < i$) from the residual vector \mathbf{r}_i . In equation (7), the coefficients β_{ij} (for $j < i$) is defined by [27]

$$\beta_{ij} = \frac{\mathbf{r}_i^H \mathbf{A} \mathbf{p}_j}{\mathbf{p}_j^H \mathbf{A} \mathbf{p}_j} \quad (8)$$

The coefficient a_i in (5) is calculated as [27]

$$a_i = \frac{\mathbf{p}_i^H \mathbf{r}_i}{\mathbf{p}_i^H \mathbf{A} \mathbf{p}_i} \quad (9)$$

3-1- Convergence analysis

The convergence of the proposed Gram-Schmit based method depends on the condition number of matrix \mathbf{A} [27]. The matrix \mathbf{A} is Hermitian Positive Definite (HPD), and its condition number is defined as follows:

$$\kappa = \frac{\lambda_{\max}(\mathbf{A})}{\lambda_{\min}(\mathbf{A})} \quad (10)$$

Where $\lambda_{\max}(\mathbf{A})$ and $\lambda_{\min}(\mathbf{A})$ are the largest and smallest eigenvalues of the matrix \mathbf{A} , respectively. Suppose that the exact solution of the linear equation (3) is $\hat{\mathbf{x}} = \mathbf{A}^{-1} \mathbf{y}_{MF}$, then it has been shown in [27] that

$$\|\hat{\mathbf{x}} - \hat{\mathbf{x}}_{i+1}\| \leq 2 \left(\frac{\sqrt{\kappa}-1}{\sqrt{\kappa}+1} \right)^i \|\hat{\mathbf{x}} - \hat{\mathbf{x}}_1\| \quad (11)$$

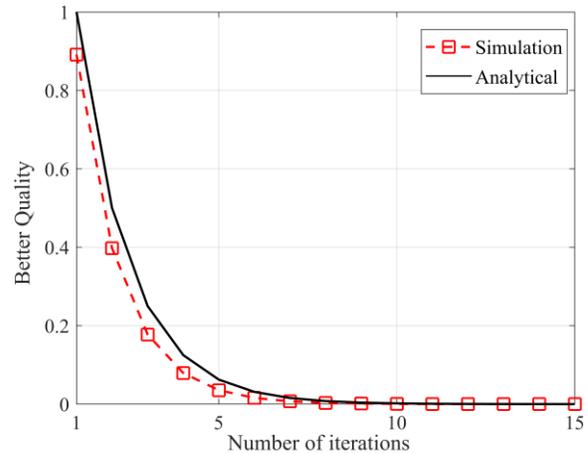


Fig. 1 Comparison between the simulation and analytical estimation error versus the number of iterations with $n_r = 128$ and $n_t = 32$.

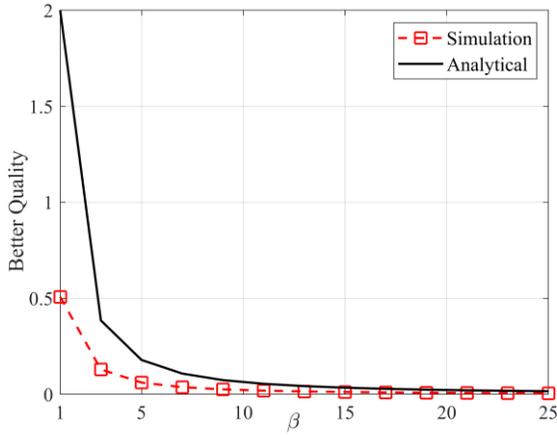


Fig. 2 Comparison between the simulation and analytical estimation error versus $\beta = \frac{n_r}{n_t}$ with $K = 3$ (3 iterations).

Where $\hat{\mathbf{x}}_1$ is the initial estimation obtained by (4). Therefore, after i iterations, the error, \mathbf{e}_{i+1} satisfies the following inequality

$$\|\mathbf{e}_{i+1}\| \leq 2 \left(\frac{\sqrt{\kappa}-1}{\sqrt{\kappa}+1} \right)^i \|\mathbf{e}_1\| \quad (12)$$

If $\left(\frac{\sqrt{\kappa}-1}{\sqrt{\kappa}+1} \right) < 1$ then based on (12), the error is decreased after each iteration and the algorithm converges.

In massive MIMO systems the largest and smallest eigenvalues of matrix \mathbf{A} can be approximated by [8]

$$\lambda_{\max}(\mathbf{A}) \approx n_t \left(1 + \frac{1}{\sqrt{\beta}} \right)^2 \quad (13)$$

and

$$\lambda_{\min}(\mathbf{A}) \approx n_t \left(1 - \frac{1}{\sqrt{\beta}} \right)^2 \quad (14)$$

Where $\beta = \frac{n_r}{n_t}$.

Using (13) and (14), the estimation error can be obtained as shown in Lemma 1.

Lemma 1. In large-Scale MIMO systems, the estimation error generated by a detector based on Gram Schmidt method at the i -th iteration can be obtained by

$$\|\mathbf{e}_{i+1}\| \leq 2 \left(\frac{n_t}{n_r} \right)^{i/2} \|\mathbf{e}_1\| \quad (15)$$

Proof: Substituting the equation (13) and (14) into the equation (10) can be rewritten as follows:

$$\kappa = \frac{(1+\sqrt{\beta})^2}{(1-\sqrt{\beta})^2} \quad (16)$$

By applying (16) in the equation (12), the inequality (15) is simply derived.

As can be seen from (15), the error is exponentially decreased when i is increased if $n_t < n_r$. The rate of the convergence depends on β . The convergence rate depends on the ratio between the number of users and number of BS antennas.

Fig. 1. shows comparison between the simulation result and analysis of the estimation error versus the number of iterations for $n_r = 128$ and $n_t = 32$. From this figure, it can be seen that the analytical error is very close to simulation error especially when the number of iterations increases. In Fig. 2, the comparison between the simulation and analysis of the estimation error versus β has been repeated, while the number of iterations is assumed to be 3. As can be seen, the distance between the simulation and analytical results is negligible especially when β increases.

4- Complexity Analysis

The computational complexity of this method can be analyzed with respect to the number of multiplications. It has been assumed that the complexity of division operation is the same as the multiplication. In this section, the order of complexity is calculated for proposed method and alternative methods. In the proposed method, we first calculate \mathbf{A} , \mathbf{r}_1 and $\hat{\mathbf{x}}_1$ in steps 1, 2 and 3 of Algorithm 1 which has the order of complexity of $O(n_t^2)$ multiplications. The number of required multiplications for K iterations of the Gram Schmidt method is $K(n_t^2 + 2n_t + K(2n_t + 1)) - n_t$ [27]. Therefore, the total complexity for the proposed algorithm is $O(Kn_t^2)$. The proposed method requires a similar number of multiplications compared to the JSDJD and GS methods in the same number iterations [16, 17], while the complexity of MMSE method is $O(n_t^3)$. Thus the proposed algorithm has lower complexity than the MMSE algorithm. Since n_t is usually large for large-scale MIMO systems, it can be observed that the proposed algorithm can evidently reduce the complexity, which it means that the proposed algorithm is suitable for large-scale MIMO systems. As mentioned before, the complexity order of the proposed method and other methods is $O(Kn_t^2)$, but as it will be shown in simulations the proposed method converges in lower number of iterations than that of other methods.

5- Simulation Result

In this section, performance of the proposed detector in massive MIMO system with 64-QAM and 16-QAM modulations have been evaluated. It is also assumed in all detection methods that the receiver knows the channel matrix completely.

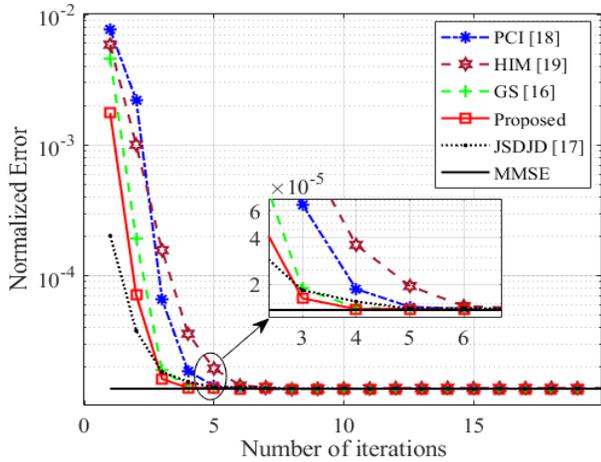


Fig. 3 Normalized estimation error versus the number of iterations for different algorithms at SNR = 15 dB with 64-QAM modulation for $n_r = 128$ and $n_t = 16$.

Fig.3. shows error of different detection methods versus number of iterations for 64-QAM modulation, when SNR is 15dB with $n_r = 128$ and $n_t = 16$. The normalized error has been defined by

$$NE = \frac{\|\mathbf{x}^* - \mathbf{x}\|}{\|\mathbf{x}\|} \quad (17)$$

Where $\|\mathbf{x}^*\|$ is the output of each method and \mathbf{x} is the exact vector of transmitted symbols.

As can be seen, since the number of BS antennas is more than the number of users, all methods converge to the estimation error of the original MMSE detector in a few number of iterations.

In Fig. 4. simulations have been repeated for $n_r = 128$ and $n_t = 64$. As it can be seen form this figure, when the

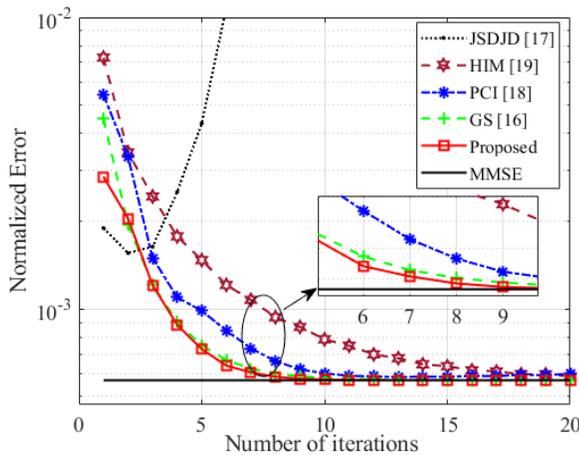


Fig. 4 Normalized estimation error versus the number of iterations for different algorithms at SNR = 15 dB with 64-QAM modulation for $n_r = 128$ and $n_t = 64$.

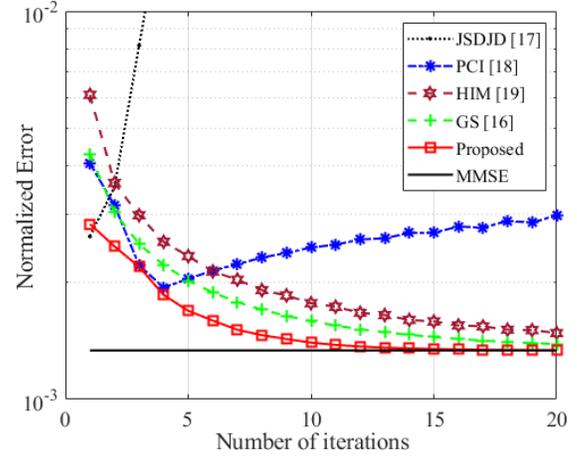


Fig. 5 Normalized estimation error versus the number of iterations for different algorithms at SNR = 15 dB with 64-QAM modulation for $n_r = 128$ and $n_t = 128$.

number of users increases, the JSDJD algorithm can not converge and performances of PCI method and HIM method are degraded. The PCI and HIM converge to the original MMSE detector after 10 and 18 iterations, respectively while our proposed and GS converge faster than PCI and HIM methods.

Fig. 5. Shows error of different detection methods versus number of iterations with $n_r = 128$ and $n_t = 128$. In this case, the JSDJD and PCI algorithms do not converge and performance of Gauss-Seidel and HIM methods are not enough close to that of MMSE detector after even 20 iterations, while performance of proposed method is very close to the MMSE detector after only 10 iterations.

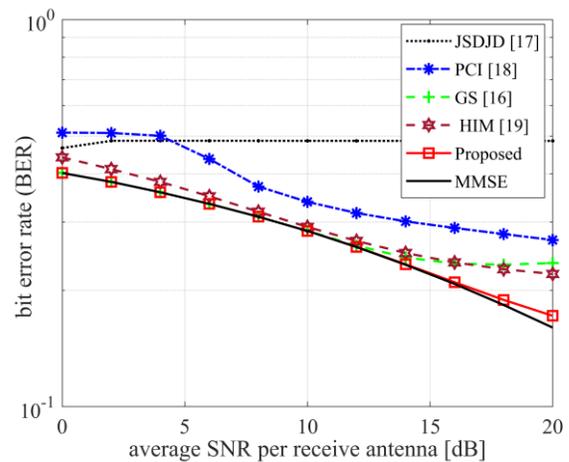


Fig. 6 BER performance comparison between the proposed and other methods in the uplink massive MIMO for 64-QAM modulation for $n_r = 128$ and $n_t = 128$ with $K = 15$ (15 iterations).

In summary, performance of the Gauss-Seidel, JSDJD, HIM and PCI methods are degraded when the number of users becomes comparable with the number of BS antennas. Even JSDJD and PCI methods do not converge, when the number of users is close to the number of BS antennas. Unlike these methods, the normalized error of the proposed algorithm converges to that of MMSE detector after a few number of iterations, even when the number of users is close to the number of BS antennas.

Fig. 6 shows BER of different detection methods versus SNR for 64-QAM modulation. In this figure, the number of base station antennas and the number of single-antenna users are considered to be $n_r = 128$ and $n_t = 128$. As it can be seen from this figure, the proposed method has a very close BER to that of original MMSE detector while the performances of Gauss-Seidel, HIM and PCI methods are not enough close to that of MMSE detector. It should be noted that, the JSDJD method does not converge at all and its BER is 1/2. As can be seen, the proposed method has about 5dB and 6dB performance improvement compared with HIM and Gauss-Seidel algorithms, respectively. In the following, all simulations have been repeated for 16-QAM modulation.

Fig.7. shows normalized error of different detection methods versus number of iterations for 16-QAM modulation when SNR is 15dB with $n_r = 128$ and $n_t = 64$. As it can be seen from this figure, the JSDJD algorithm can not converge to the estimation error of the original MMSE detector.

The HIM and Gauss-Seidel methods converge to the original MMSE detector after 17 and 11 iterations, respectively. Also, performance of PCI method is not enough close to that of MMSE detector after 20 iterations, while performance of proposed method is very close to the MMSE detector after only 9 iterations.

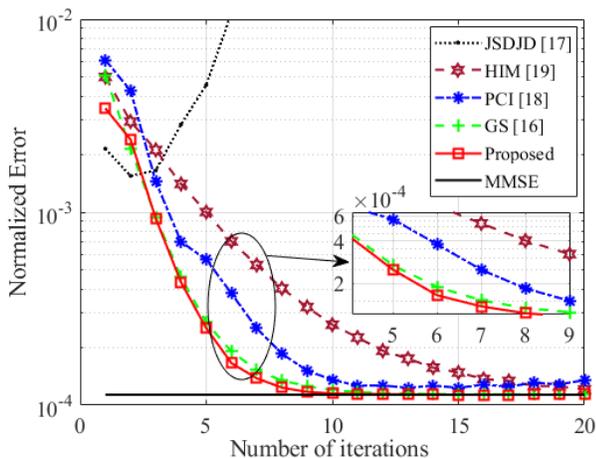


Fig. 7 Normalized estimation error versus the number of iterations for different algorithms at SNR = 15 dB with 16-QAM modulation for $n_r = 128$ and $n_t = 64$.

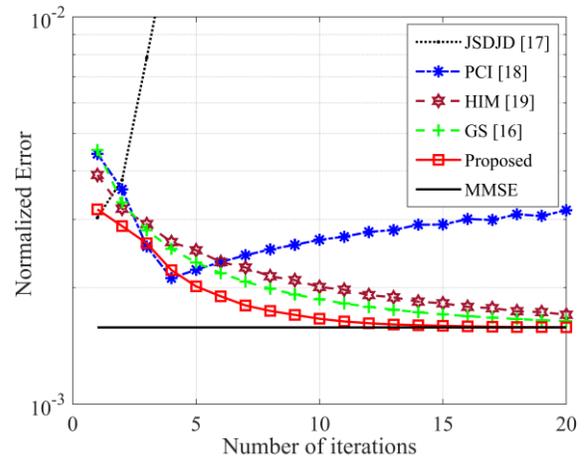


Fig. 8 Normalized estimation error versus the number of iterations for different algorithms at SNR = 15 dB with 16-QAM modulation for $n_r = 128$ and $n_t = 128$.

In Fig. 8 simulations have been repeated for $n_r = 128$ and $n_t = 128$. In this case, the JSDJD and PCI algorithms do not converge to the normalized error of the original MMSE detector and performance of Gauss-Seidel method is not enough close to that of MMSE detector after 20 iterations, while performance of proposed method is very close to the MMSE detector after only 10 iterations. Also, performance of HIM method is not enough close to that of MMSE detector even after 20 iterations.

As it can be seen from this figure, when the number of users increases, the JSDJD algorithm can not converge to the estimation error of the original MMSE detector and performance of PCI method and HIM method are degraded. The PCI and HIM converge to the original MMSE detector after 10 and 18 iterations, respectively.

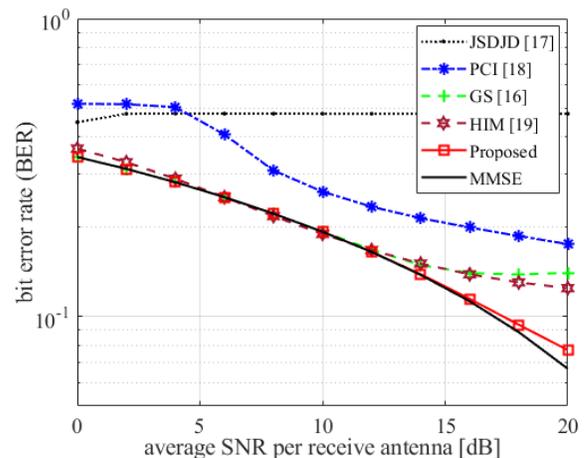


Fig. 9 BER performance comparison between the proposed and other methods in the uplink massive MIMO for 16-QAM modulation for $n_r = 128$ and $n_t = 128$ with $K = 15$ (15 iterations).

Fig. 9 shows BER of different detection methods versus SNR for 16-QAM modulation. In this figure, the number of base station antennas and the number of single-antenna users are considered to be $n_r = 128$ and $n_t = 128$. As it can be seen from this figure, the proposed method is able to converge to the performance of the original MMSE detector, while the performances of Gauss-Seidel, HIM and PCI methods are not enough close to that of MMSE detector. It should be noted that, the JSDJD method does not converge and its BER is 1/2. As can be seen, the proposed method has about 5dB and 6dB performance improvement compared with HIM and Gauss-Seidel algorithms, respectively.

It was demonstrated that the proposed method always converges to the original MMSE detector even when the number of users is very close to the number of BS antennas. But the proposed method has a disadvantage. The main disadvantage is that when the number of users is close to the number of BS antennas (it is sometimes called loaded scenario), the proposed algorithm needs a high number of iterations to converge. This leads to the increase of the complexity. In future works, we try to modify the proposed method to accelerate its convergence in loaded scenario.

6- Conclusions

In this paper, a novel low complexity iterative detection algorithm for multiuser massive MIMO uplink detection without complicated matrix inversion was proposed. It was shown that the proposed method always converges to the original MMSE detector even when the number of users is very close to the number of BS antennas and its performance is very close to the original near optimum MMSE detector.

References

- [1] J. Thompson, X. Ge, H.-C. Wu, R. Irmer, H. Jiang, G. Fettweis, et al., "5G wireless communication systems: Prospects and challenges," *IEEE Communications Magazine*, vol. 52, pp. 62-64, 2014.
- [2] E. Björnson, E. G. Larsson, and T. L. Marzetta, "Massive MIMO: Ten myths and one critical question," *IEEE Communications Magazine*, vol. 54, pp. 114-123, 2016.
- [3] M. A. Albreem, M. Juntti, and S. Shahabuddin, "Massive MIMO detection techniques: a survey," *IEEE Communications Surveys & Tutorials*, vol. 21, pp. 3109-3132, 2019.
- [4] P. Zhang, X. Yang, J. Chen, and Y. Huang, "A survey of testing for 5G: Solutions, opportunities, and challenges," *China Communications*, vol. 16, pp. 69-85, 2019.
- [5] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. Soong, et al., "What will 5G be?," *IEEE Journal on Selected areas in Communications*, vol. 32, pp. 1065-1082, 2014.
- [6] Y. Arjouni and S. Faruque, "Artificial Intelligence for 5G Wireless Systems: Opportunities, Challenges, and Future Research Direction," in 2020 10th Annual Computing and Communication Workshop and Conference (CCWC), 2020, pp. 1023-1028.
- [7] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Communications Magazine*, vol. 52, pp. 186-195, 2014.
- [8] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, et al., "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Processing Magazine*, vol. 30, pp. 40-60, 2013.
- [9] M. O. Damen, H. El Gamal, and G. Caire, "On maximum-likelihood detection and the search for the closest lattice point," *IEEE Transactions on Information Theory*, vol. 49, pp. 2389-2402, 2003.
- [10] C. Thrampoulidis, W. Xu, and B. Hassibi, "Symbol error rate performance of box-relaxation decoders in massive MIMO," *IEEE Transactions on Signal Processing*, vol. 66, pp. 3377-3392, 2018.
- [11] E. G. Larsson and J. Jalden, "Fixed-complexity soft MIMO detection via partial marginalization," *IEEE transactions on Signal Processing*, vol. 56, pp. 3397-3407, 2008.
- [12] Z. Guo and P. Nilsson, "Algorithm and implementation of the K-best sphere decoding for MIMO detection," *IEEE Journal on Selected areas in Communications*, vol. 24, pp. 491-503, 2006.
- [13] N. A. Khan, S. Ahmed, D. M. S. Bhatti, M. Z. Tunio, and S. Kim, "Study of MIMO detection schemes for emerging wireless communications," *International Journal of Computer Science and Network Security*, vol. 18, pp. 142-149, 2018.
- [14] Y. N. Ahmed, "Enhanced low-complexity matrix inversion method for Massive MIMO systems," in 2019 16th International Symposium on Wireless Communication Systems (ISWCS), 2019, pp. 495-499.
- [15] C. Tang, Y. Tao, Y. Chen, C. Liu, L. Yuan, and Z. Xing, "Approximate iteration detection and precoding in Massive MIMO," *China Communications*, vol. 15, pp. 183-196, 2018.
- [16] L. Dai, X. Gao, X. Su, S. Han, I. Chih-Lin, and Z. Wang, "Low-complexity soft-output signal detection based on Gauss-Seidel method for uplink multiuser large-scale MIMO systems," *IEEE Transactions on Vehicular Technology*, vol. 64, pp. 4839-4845, 2015.
- [17] X. Qin, Z. Yan, and G. He, "A near-optimal detection scheme based on joint steepest descent and Jacobi method for uplink Massive MIMO systems," *IEEE Communications Letters*, vol. 20, pp. 276-279, 2016.
- [18] G. Peng, L. Liu, P. Zhang, S. Yin, and S. Wei, "Low-computing-load, high-parallelism detection method based on Chebyshev iteration for Massive MIMO systems with VLSI architecture," *IEEE Transactions on Signal Processing*, vol. 65, pp. 3775-3788, 2017.
- [19] F. Jin, Q. Liu, H. Liu, and P. Wu, "A Low Complexity Signal Detection Scheme Based on Improved Newton

- Iteration for Massive MIMO Systems," *IEEE Communications Letters*, vol. 23, pp. 748-751, 2019.
- [20] M. Wu, B. Yin, G. Wang, C. Dick, J. R. Cavallaro, and C. Studer, "Large-scale MIMO detection for 3GPP LTE: Algorithms and FPGA implementations," *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, pp. 916-929, 2014.
- [21] D. Zhu, B. Li, and P. Liang, "On the matrix inversion approximation based on Neumann series in massive MIMO systems," in *2015 IEEE International Conference on Communications (ICC)*, 2015, pp. 1763-1769.
- [22] X. Liu, Z. Zhang, X. Wang, J. Lian, and X. Dai, "A Low complexity high performance weighted Neumann series-based Massive MIMO detection," in *2019 28th Wireless and Optical Communications Conference (WOCC)*, 2019, pp. 1-5.
- [23] M. A. Albreem, "Approximate matrix inversion methods for massive mimo detectors," in *2019 IEEE 23rd International Symposium on Consumer Technologies (ISCT)*, 2019, pp. 87-92.
- [24] M. A. Albreem, A. A. El-Saleh, and M. Juntti, "Linear Massive MIMO uplink detector based On joint jacobi and Gauss-Seidel methods," in *2020 16th International Conference on the Design of Reliable Communication Networks DRCN 2020*, 2020, pp. 1-4.
- [25] M. Burger, B. Kaltenbacher, and A. Neubauer, "Iterative solution methods," in *Handbook of Mathematical Methods in Imaging*, ed: Springer, 2011, pp. 345-384.
- [26] R. I. Hernández, R. Baghaie, and K. Kettunen, "Implementation of Gram-Schmidt conjugate direction and conjugate gradient algorithms," in *Proc. IEEE Finish Signal Processing Symposium*, 1999, pp. 165-169.
- [27] J. R. Shewchuk, "An introduction to the conjugate gradient method without the agonizing pain," ed: Carnegie-Mellon University. Department of Computer Science, 1994.

Mojtaba Amiri received the B.Sc. degree in electrical engineering from Shiraz University of Technology, Iran, in 2015. He received the M.Sc. degree in electrical engineering from Shahed University, Iran, in 2018. His research interests include signal detection in large-scale MIMO systems, direction of arrival (DoA) estimation, and compressive sensing (CS).

Mahmoud Ferdosizade Naeiny was born in Iran, in 1979. He received the B.Sc. degree (Hons.) from the Amirkabir University of Technology (Tehran Polytechnic) in 2002 and the M.Sc. and Ph.D. degrees from the Sharif University of Technology in 2004 and 2010, respectively. He is currently an associate professor with Shahed University. His research interests include array signal processing, compressive sensing, design and implementation of OFDM systems including the synchronization issues, software-defined radio, MIMO systems and the application of signal processing techniques in communications. He has been the Chair **with** the Institute of Modern Technologies on Information and Communications, Shahed University, since 2014.