Nonlinear State Estimation Using Hybrid Robust Cubature Kalman Filter

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Abstract

In this paper, a novel filter is provided that estimates the states of any nonlinear system, both in the presence and absence of uncertainty with high accuracy. It is well understood that a robust filter design is a compromise between the robustness and the estimation accuracy. In fact, a robust filter is designed to obtain an accurate and suitable performance in presence of modelling errors. So in the absence of any unknown or time-varying uncertainties, the robust filter does not provide the desired performance. The new method provided in this paper, which is named hybrid robust cubature Kalman filter (CKF), is constructed by combining a traditional CKF and a novel robust CKF. The novel robust CKF is designed by merging a traditional CKF with an uncertainty estimator so that it can provide the desired performance in the presence of uncertainty. Since the presence of uncertainty results in a large innovation value, the hybrid robust CKF adapts itself according to the value of the normalized innovation. The CKF and robust CKF filters are run in parallel and at any time, a suitable decision is taken to choose the estimated state of either the CKF or the robust CKF as the final state estimation. To validate the performance of the proposed filters, two examples are given that demonstrate their promising performance.

Keywords: Uncertainty; State Estimation; Cubature Kalman Filter (CKF); Robust CKF; Hybrid Robust CKF.

1. Introduction

One of the essential problems in control theory and signal processing is the problem of dynamic state estimation. Mostly, the extended Kalman filter (EKF) is used to estimate the states of nonlinear systems [1,2]. In EKF, it is necessary to calculate Hessian and Jacobian matrices at each iteration. Therefore, this filter has linearization error and is not suitable for highly nonlinear functions. Cubature Kalman filter (CKF) has been presented to dominate the limitations of EKF [3-5]. Moreover, the CKF has a higher approximation accuracy compared to the EKF and has an easier algorithm to implement, because it avoids weighty calculation of the Jacobian and Hessian matrices. Because of these advantages, CKF has been used in many applications [6-8]. When the process and measurement models of a system are known, we will have the best performance for each estimator (EKF or CKF). Nevertheless, in many physical systems, the obtained model is an approximate with parametric uncertainty and unknown external input. Furthermore, the process and measurement noises may be colored and biased instead of being zero mean and white. Recently, uncertainties in the plant model are considered and several robust filtering methods have been provided, including robust Kalman filter [9,10], robust minimum variance filters [11,12], smooth variable structure filter [13], risk-sensitive filter [14] and so on. These filters are different in terms of the uncertainty present in the plant. Unfortunately, finding an effective and accurate model for the plant in the presence of uncertainty is often difficult.

The present work solves this problem and acts in such a way that does not require any specific information about the plant uncertainty. The proposed modification in CKF is capable of preserving filter performance in the presence of uncertainty and unknown disturbance in the plant description. In other words, a new robust nonlinear filter will be proposed. In addition, these uncertainties do not have an upper limit (like a bounded norm).

It is well understood that a robust filter design is a compromise between estimation accuracy and robustness. A robust filter is designed to obtain an accurate and suitable performance considering the errors of modelling [15-17]. Thus, in the absence of any unknown or time-varying uncertainties, the robust filter does not provide the desired performance. This issue has caused that in this paper, a hybrid robust CKF is proposed so that detects and adjusts itself with respect to the uncertainty of the system. The hybrid robust CKF consists of both filters: a CKF and a robust CKF that run in parallel. At each time step, choosing one of these two types of filters to estimate the final state is related to the uncertainty and based on the amount of the normalized innovation corresponding to the two filters (CKF and robust CKF). Therefore, it is expected that the hybrid robust CKF has a desirable performance even in the absence of uncertainty.

This paper is configured as follows. In Section 2, the problem formulation is briefly described. The estimation

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of uncertainty using a low pass filter is investigated in Section 3. Section 4 and Section 5 include the proposed robust CKF and hybrid robust CKF, respectively. In Section 6, simulation results and comparison of the proposed algorithms with CKF algorithm are given. Finally, in Section 7, conclusion is presented.

2. Problem Formulation

Consider the discrete-time nonlinear stochastic control system.

$$x_k = f(x_{k-1}, u_{k-1}) + w_{k-1} \quad (1)$$

$$z_k = h(x_{k-1}) + v_k \quad (2)$$

Where $x_k \in R^n$ is the stochastic state vector, $u_k \in R^p$ is a known control input and $z_k \in R^m$ is the measurement vector, w_{k-1} and v_k are zero mean-white Gaussian noises with covariance matrices Q_{k-1} and R_k, respectively. The process noise w_{k-1} denotes any kind of uncertainty which disturbs the system. Moreover, w_{k-1} and v_k are assumed to be uncorrelated. The problem is to estimate the system states using the measurements z_k.

3. Estimation of Uncertainty Using a Low Pass Filter

As said before, the process noise w_{k-1} represents any kind of uncertainty which disturbs the nominal system $x_k = f(x_{k-1}, u_{k-1})$. In the state transition equation (1), a quantity equal to the uncertainty w_{k-1} is expressed as

$$w_{eq,k-1} = x_k - f(x_{k-1}, u_{k-1}) \quad (3)$$

Then, an estimate of this uncertainty at the existing step is written as follows

$$\hat{w}_k = \hat{w}_{eq,k-1}(\hat{x}_k) = \hat{x}_k - f(\hat{x}_{k-1}, u_{k-1}) \quad (4)$$

To estimate the uncertainty, a low pass filter is used as [18].

$$\hat{w}_k = \prod \hat{w}_{eq,k-1}(\hat{x}_k) = \prod (\hat{x}_k - f(\hat{x}_{k-1}, u_{k-1})) \quad (5)$$

where Π is a diagonal matrix with low pass filter entries. While a lot of candidates are possible in the selection of the Π matrix; here, the following first-order discrete filters have been applied.

$$k_i(z) = \frac{b_i}{1 - a_i z^{-1}} \qquad i = 1,2,\ldots,n \quad (6)$$

Therefore, the matrix Π will take the form

$$\prod(z) = diag(k_1, k_2, \ldots, k_n) \quad (7)$$

By expanding (7), the state-space realization of (5) is written as follows

$$\hat{w}_k = A\hat{w}_{k-1} + B(\hat{x}_k - f(\hat{x}_{k-1}, u_{k-1})) \quad (8)$$

where, $A = diag(a_1, \ldots, a_n)$ and $B = diag(b_1, \ldots, b_n)$. The discrete LPFs in (6) should have a unity DC gain, that is $k_1(1) = 1 (i = 1, \ldots, n)$. Thus, $a_1 + b_1 = 1$ and naturally $A + B = I$.

4. Robust Cubature Kalman Filter

In this section, the cubature Kalman filter is introduced briefly and then it is extended to develop a new robust cubature Kalman filter.

4.1 Cubature Kalman Filter

All of known filters have problems such as divergence in high-dimensions. Dimension problem is related to state space and is observed in higher dimension state space models. In recent years, the cubature Kalman filter was proposed in order to solve divergence and dimension problems [3]. This filter is a nonlinear method to estimate the system states with an upper dimension limit based on the spherical-radial cubature rule [4]. There is no need to make a derivative in the cubature rule. Thus, it is not necessary to compute the Jacobian and Hessian matrices. CKF algorithm is demonstrated in [3,4] completely.

4.2 Robust Cubature Kalman Filter

The robust CKF is a combination of the traditional CKF with a new uncertainty estimator. This filter is designed to estimate the states of any nonlinear system in presence of various types of uncertainties including, the parameter uncertainty or the unknown input. This filtering algorithm is similar to the traditional CKF, except that the estimated uncertainty $\hat{w}_{k-1}$ and an uncertainty estimation error covariance $P_{w,k-1}$ are included in the algorithm. The robust CKF consists of the following steps.

1. Initialize ($\hat{x}_{k-1\mid k-1}$ and $p_{k-1\mid k-1}$)
2. Initial density at time $k-1$ can be decomposed as follows

$$P_{k-1|k-1} = S_{k-1|k-1} S_{k-1|k-1}^T \quad (9)$$

3. Obtain the cubature points

$$X_{i,k-1|k-1} = S_{k-1|k-1}\xi_i + \hat{x}_{k-1|k-1} \quad \begin{pmatrix} i = 1,2,\ldots,m \\ m = 2n \end{pmatrix} \quad (10)$$

4. In this step, the cubature points pass through the nonlinear function $f(.)$.

$$X_{i,k|k-1} = f(X_{i,k-1|k-1}, u_{k-1}) + \hat{w}_{k-1} \quad (11)$$

5. The predicted mean is computed at the time update

$$\hat{x}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} X_{i,k|k-1} \quad (12)$$

6. The predicted covariance is computed at the time update

$$P_{k|k-1}=\frac{1}{m}\sum_{i=1}^{m}X_{i,k|k-1}X_{i,k|k-1}^{T}-\hat{x}_{k|k-1}\hat{x}_{k|k-1}^{T}+P_{w,k} \quad (13)$$

7. The covariance matrix achieved from the previous step is decomposed again as

$$P_{k|k-1}=S_{k|k-1}S_{k|k-1}^{T} \quad (14)$$

8. Obtain the new cubature points

$$X_{i,k|k-1}=S_{k|k-1}\xi_i+\hat{x}_{k|k-1} \quad (15)$$

9. The cubature points pass through the nonlinear function $h(.)$.

$$Y_{i,k|k-1}=h(X_{i,k|k-1},u_{k-1}) \quad (16)$$

10. The predicted observation is computed at the measurement update

$$\hat{y}_{k|k-1}=\frac{1}{m}\sum_{i=1}^{m}Y_{i,k|k-1} \quad (17)$$

11. The predicted innovation covariance is calculated at the measurement update

$$P_{yy,k|k-1}=\frac{1}{m}\sum_{i=1}^{m}Y_{i,k|k-1}Y_{i,k|k-1}^{T}-\hat{y}_{k|k-1}\hat{y}_{k|k-1}^{T}+R_k \quad (18)$$

12. The predicted cross covariance is

$$P_{xy,k|k-1}=\frac{1}{m}\sum_{i=1}^{m}X_{i,k|k-1}Y_{i,k|k-1}^{T}-\hat{x}_{k|k-1}\hat{y}_{k|k-1}^{T} \quad (19)$$

13. The CKF gain is

$$K_k=P_{xy,k|k-1}P_{yy,k|k-1}^{-1} \quad (20)$$

14. Compute the updated state and covariance

$$\hat{x}_{k|k}=\hat{x}_{k|k-1}+K_k(y_k-\hat{y}_{k|k-1}) \quad (21)$$

$$P_{k|k}=P_{k|k-1}-K_kP_{yy,k|k-1}K_k^{T} \quad (22)$$

15. Update the uncertainty estimation

$$\hat{w}_k=A\hat{w}_{k-1}+B(\hat{x}_k-f(\hat{x}_{k-1},u_{k-1})) \quad (23)$$

16. Update the uncertainty estimation error covariance (derived in lemma 1)

$$P_{w,k}=P_{w,k-1}+(BK_k)R(BK_k)^{T}+Q_{k-1} \quad (24)$$

Block diagram of the proposed robust CKF is shown in Figure 1.

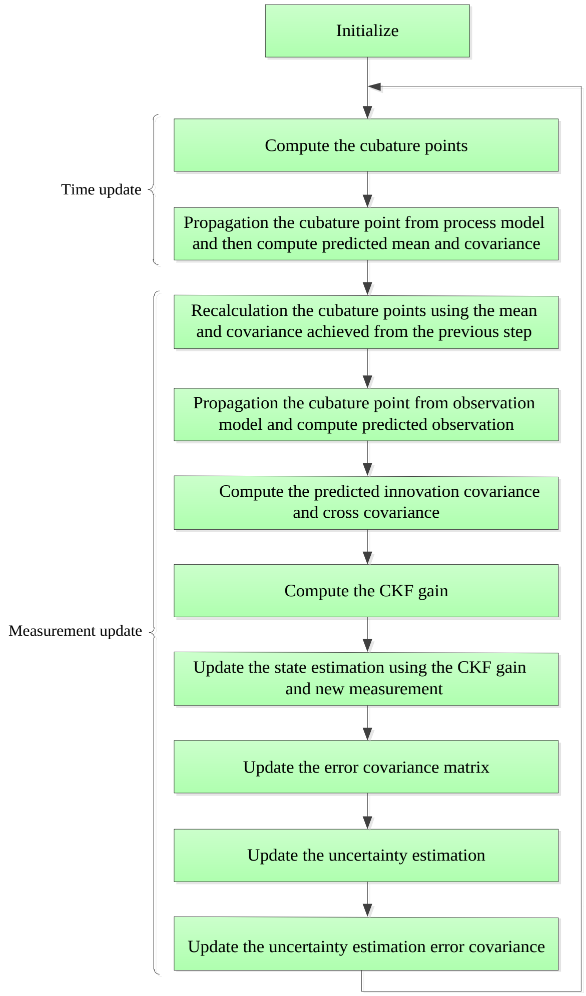

Fig. 1. Block diagram of the proposed robust CKF.

Lemma 1. The uncertainty estimation error covariance can be written as follows

$$P_{w,k}=P_{w,k-1}+(BK_k)R(BK_k)^{T}+Q_{k-1} \quad (25)$$

Proof: We have

$$\hat{x}_{k|k-1}=f(\hat{x}_{k-1|k-1},u_{k-1})+\hat{w}_{k-1} \quad (26)$$

Using this equation, the estimated uncertainty in (8) can be obtained as follows

$$\begin{aligned}&\hat{w}_k=A\hat{w}_{k-1}+B(\hat{x}_k-f(\hat{x}_{k-1},u_{k-1}))=\\&A\hat{w}_{k-1}+B(\hat{x}_k-\hat{x}_{k|k-1})+B\hat{w}_{k-1}=\hat{w}_{k-1}+B(\hat{x}_k-\hat{x}_{k|k-1})\end{aligned} \quad (27)$$

Since $A+B=I$.

From equation (21) we have

$$\hat{x}_{k|k}-\hat{x}_{k|k-1}=K_k(y_k-\hat{y}_{k|k-1}) \quad (28)$$

Now, using (27) and (28) we obtain

$$\hat{w}_k = \hat{w}_{k-1} + B(\hat{x}_k - \hat{x}_{k|k-1}) = \hat{w}_{k-1} + BK_k(y_k - \hat{y}_{k|k-1}) \quad (29)$$

Also, we can write

$$w_k = w_{k-1} + w_k - w_{k-1} = w_{k-1} + \Delta w_k \quad (30)$$

where, Δw_k is the difference in uncertainty between the $k-1$th and kth instant and is assumed to be white Gaussian with zero mean and covariance Q_{k-1}.

Subtracting (29) from (30), gives

$$(w_k - \hat{w}_k) = (w_{k-1} - \hat{w}_{k-1}) - BK_k(y_k - \hat{y}_{k|k-1}) + \Delta w_k \quad (31)$$

$$\tilde{w}_k = \tilde{w}_{k-1} - BK_k\tilde{y}_k + \Delta w_k \quad (32)$$

where, $(y_k - \hat{y}_{k\,|\,k-1})$ can be approximated by the measurement noise v_k. Noises $\tilde{w}_k$, v_k and Δw_k are uncorrelated and the uncertainty error covariance is as follows.

$$E[\tilde{w}_k\tilde{w}_k^T] = P_{w,k} \quad (33)$$

Finally, using equations (32) and (33), the uncertainty estimation error covariance, $p_{w,k}$ can be derived as

$$P_{w,k} = P_{w,k-1} + (BK_k)R(BK_k)^T + Q_{k-1} \quad (34)$$

5. Hybrid Robust Cubature Kalman Filter

A robust filter will be designed in this section to keep a balance between uncertainty and estimation error; therefore, it has a desired performance in the presence or absence of any unknown input or uncertainty In fact, the CKF and robust CKF run in parallel in the proposed method. At any instant, a proper decision is taken to choose either the estimated state of the CKF or the robust CKF as the final state estimation.

The presence of uncertainty shows itself as a big innovation. A simple assessment method for this problem is based on the normalized innovation which is given as

$$\varepsilon_k = \lambda_k^T \delta_k \lambda_k \quad (35)$$

where, λ_k is the innovation and δ_k is the innovation covariance. At each time step, ε_k is computed by both of the filters. Finally, the filter is selected as the final state estimator that has less innovation. Instead of a statistical decision making, a moving average of the normalized innovations is considered as follows

$$\varepsilon_k^s = \sum_{i=k-s+1}^{k} \varepsilon_k(i) \quad (36)$$

where s is the moving window's width. Suppose at time k, the CKF state estimation is $x_{ckf}(k|k)$ and covariance matrix is $p_{ckf}(k|k)$, the robust CKF state estimation is $x_{Rckf}(k|k)$ and covariance matrix is $p_{Rckf}(k|k)$ and the corresponding amounts for the hybrid robust CKF are $x_{HyRckf}(k|k)$ and $p_{HyRckf}(k|k)$, respectively. The output of robust CKF is one of the two types of the CKF or the robust CKF. However, as stated above, the output of this filter is the output of the filter (CKF or robust CKF) that has less innovation. The innovation of the CKF and the robust CKF are demonstrated by $\varepsilon_{k,ckf}^s$ and $\varepsilon_{k,Rckf}^s$, respectively. In this type of filter, decision-making is performed as follows

$$[x_{ckf}(k\,|\,k),\ P_{ckf}(k\,|\,k)] = CKF[x_{ckf}(k-1\,|\,k-1),\ P_{ckf}(k-1\,|\,k-1)]$$

$$[x_{Rckf}(k\,|\,k),\ P_{Rckf}(k\,|\,k)] = ROBUSTCKF[x_{Rckf}(k-1\,|\,k-1),\ P_{Rckf}(k-1\,|\,k-1)]$$

$$if\ \ \varepsilon_{k,ckf}^s > \gamma\varepsilon_{k,Rckf}^s$$

$$[x_{HyRckf}(k\,|\,k),\ P_{HyRckf}(k\,|\,k)] = [x_{Rckf}(k\,|\,k),\ P_{Rckf}(k\,|\,k)]$$

$$else$$

$$[x_{HyRckf}(k\,|\,k),\ P_{HyRckf}(k\,|\,k)] = [x_{ckf}(k\,|\,k),\ P_{ckf}(k\,|\,k)]$$

$$end$$

where, γ is a scalable parameter that depends on the permissible amount of uncertainty in the system.

Block diagram of the proposed hybrid robust CKF is shown in Figure 2.

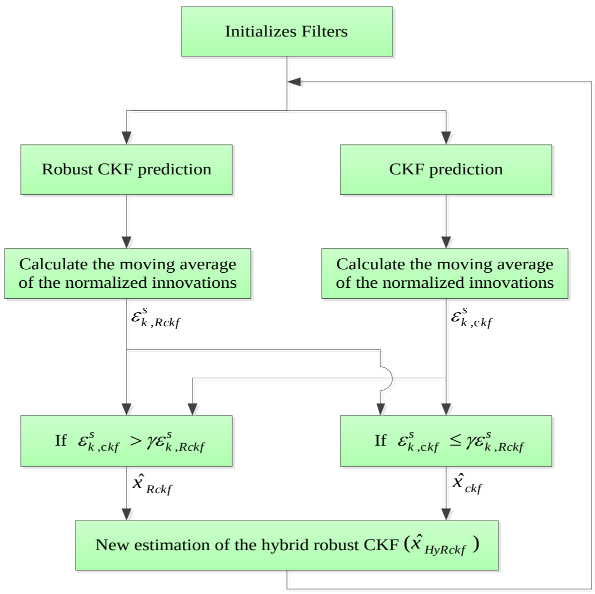

Fig. 2. Block diagram of the proposed hybrid robust CKF.

6. Simulation Results

In this section, two examples are given to show the performance of the proposed filters, in comparison with the traditional CKF. The first example is related to a ballistic target motion model with unknown ballistic coefficient and aerodynamic forces adopted from [19]. The second example is related to the Euler-discretized van der Pol oscillator that is adopted from [20].

Example 1) The ballistic target motion model with unknown ballistic coefficient is given by [19].

$$s_{k+1} = \Phi s_k + Gf(s_k) + G\begin{bmatrix} 0 \\ -g \end{bmatrix} + w_k \tag{37}$$

where s_k is the target state vector given as follows

$$s_k = \begin{bmatrix} x_k & \dot{x}_k & y_k & \dot{y}_k & \beta_k \end{bmatrix} \tag{38}$$

x_k and y_k are target positions, $\dot{x}_k$ target velocity along x axis, and $\dot{y}_k$ target velocity along y axis and B_k is the unknown ballistic coefficient that evolves through time as follows.

$$\beta_k = \beta_{k-1} + w_k^\beta \tag{39}$$

where w_k^β is a sequence of independent, identically distributed (IID) Gaussian variables with zero mean and variance $\tilde{q}$. Furthermore, g is the gravity acceleration and the matrices Φ and G are as follows

$$\Phi = \begin{bmatrix} 1 & \Delta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} \frac{\Delta^2}{2} & 0 \\ \Delta & 0 \\ 0 & \frac{\Delta^2}{2} \\ 0 & \Delta \\ 0 & 0 \end{bmatrix} \tag{40}$$

where Δ is the time interval between two consecutive radar measurements. w_k is a sequence of IID Gaussian random vectors, with zero mean and a covariance matrix as follows.

$$Q = \begin{bmatrix} \frac{q\Delta^3}{3} & \frac{q\Delta^2}{2} & 0 & 0 & 0 \\ \frac{q\Delta^2}{2} & q\Delta & 0 & 0 & 0 \\ 0 & 0 & \frac{q\Delta^3}{3} & \frac{q\Delta^2}{2} & 0 \\ 0 & 0 & q\Delta^2 & q\Delta & 0 \\ 0 & 0 & 0 & 0 & \tilde{q}\Delta \end{bmatrix} \tag{41}$$

where q is a positive real number and $\tilde{q}$ is the variance of w_k^β in (39). Finally, $f(s_k)$ is the nonlinear function in (37) that denotes the ballistic coefficient β_k and is given by

$$f(s_k) = -0.5\frac{g}{\beta_k}\rho(y_k)\sqrt{\dot{x}_k^2 + \dot{y}_k^2}\begin{bmatrix} \dot{x}_k \\ \dot{y}_k \end{bmatrix} \tag{42}$$

where $p(.)$ is the air density, which is defined as follows.

$$\rho(y_k) = c_1 \exp(-c_2 y) \tag{43}$$

with

$$\begin{cases} c_1 = 1.227, \; c_2 = 1.093\times 10^{-4} & for \;\; y < 9144m \\ c_1 = 1.754, \; c_2 = 1.49\times 10^{-4} & for \;\; y \geq 9144m \end{cases}.$$

The two dimensional observation vector is as $z_k = [r_k \quad \varepsilon_k]^T$, where r_k is the measured range and ε_k is the elevation angle. Measurement equation is expressed as follows

$$z_k = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \arctan(\frac{y_k}{x_k}) \end{bmatrix} + v_k \tag{44}$$

v_k is a Gaussian random sequence with zero mean and covariance matrix

$$R = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{bmatrix} \tag{45}$$

To simulate the trajectory, the parameter values are chosen as $g = 9.8$, $\Delta = 2s$, $q = \tilde{q} = 5$, $\sigma_r = 150m$ $\sigma_r = 150m$ and $\sigma_\varepsilon = 150rad$. All three filters are initialized as follows

$$x_0 = \hat{x}_{0|0} = \begin{bmatrix} 232000m \\ 2290\cos(190^\circ)m/s \\ 88000m \\ 2290\sin(190^\circ)m/s \\ 40000kg.m^{-1}.s^{-2} \end{bmatrix}$$

$$P_{0|0} = diag([1000^2m^2, 50^2m^2.s^{-2}, 1000^2m^2, 50^2m^2.s^{-2}, 2500^2kg^2.m^{-2}.s^{-4}])$$

To evaluate the performance of the proposed filters, two cases are considered:

A) It is assumed that the ballistic coefficient and aerodynamics forces are completely unknown, so instead of $f(.)$ in (37) zero is replaced.

B) It is assumed that the ballistic coefficient and aerodynamics forces are completely known.

For the robust CKF, the parameters are selected as $A = 0.8I_{5\times5}$, $B = 0.2I_{5\times5}$ initial mean of uncertainty is $\hat{w}_0 = [0 \quad 0 \quad 0 \quad 0 \quad 0]^T$ and initial uncertainty covariance is assumed to be $P_{w,0} = Q$. In addition, the scaling parameter and the moving window width are $\gamma = 2.5$ and $s = 5$ respectively. The root mean square error (RMSE) index is used to compare the performance of the three filters. This index is defined as

$$RMSE(x) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \hat{x}_i)^2} \quad (46)$$

For all of the three filters, the corresponding root mean square error (RMSE) curves for the estimated target position are shown in Figs (3-6). These figures have been achieved through the implementation of 100 Monte Carlo runs. It can be seen in these figures that when the ballistic coefficient and aerodynamics forces are unknown (figs 3-4), the robust CKF and hybrid robust CKF give better results than the CKF. But when the ballistic coefficient and aerodynamics forces are known (figs 5-6), the hybrid CKF follows the traditional CKF and the performance of these two filters is better than the robust CKF. So, both in presence and in the absence of any uncertainty, the hybrid CKF has a promising performance.

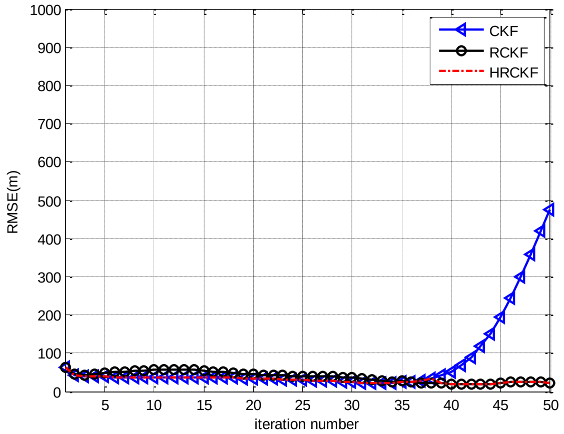

Fig. 3. RMS error along x-axis when the ballistic coefficient and aerodynamics forces are unknown.

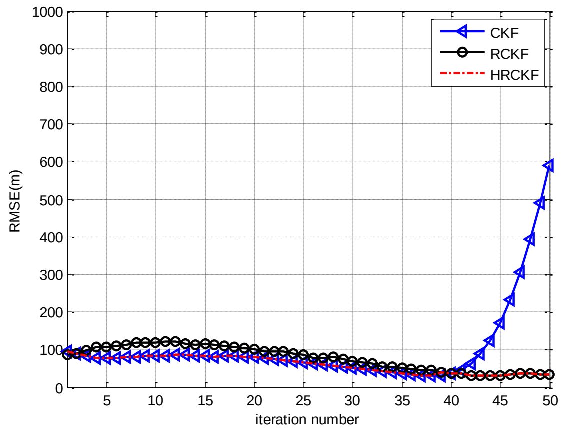

Fig. 4. RMS error along y-axis when the ballistic coefficient and aerodynamics forces are unknown.

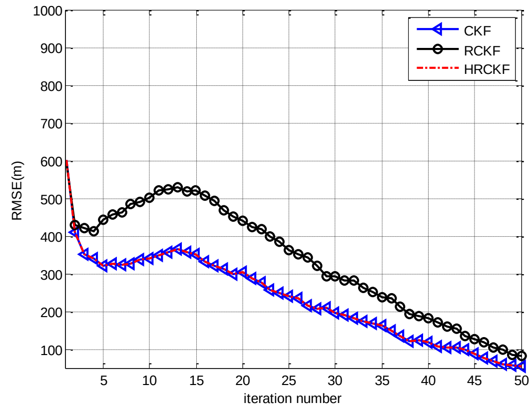

Fig. 5. RMS error along x-axis when the ballistic coefficient and aerodynamics forces are known.

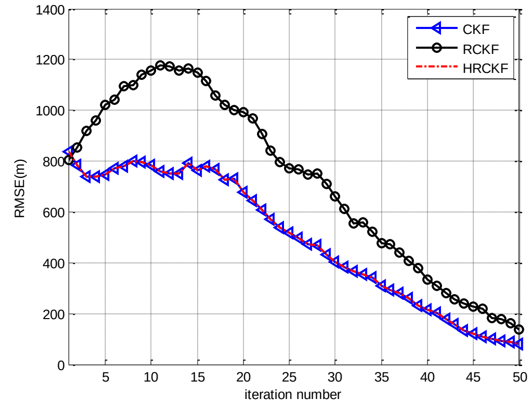

Fig. 6. RMS error along y-axis when the ballistic coefficient and aerodynamics forces are known.

Example 2) The Euler-discretized van der Pol oscillator is given as follows [20].

$$\begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} = \begin{bmatrix} x_{1,k-1} + Tx_{2,k-1} \\ -Tx_{1,k-1} + (T + 1 - Tx_{1,k-1}^2)x_{2,k-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_{k-1} + w_{k-1} \quad (47)$$

where, $x_k = [x_{1,k} \quad x_{2,k}]^T$ is the state vector, u_{k-1} is the external input assumed to be unknown, w_{k-1} is a white Gaussian noise with zero mean and covariance $Q = 10^{-6} I_{2\times2}$ and T is the sampling interval. Measurement equation is given as follows

$$z_k = [1 \quad 1] x_k + v_k \quad (48)$$

The measurement noise v_k is a zero mean white Gaussian with covariance $R = 0.04$. The input u_{k-1} is given by.

$$u_{k-1} = \begin{cases} T\sin(2kT), & if \quad kT < 10s \; or \; kT > 30s \\ T\sin(2kT) + 0.5, & if \quad 10s < kT < 20s \\ T\sin(2kT) - 0.5, & if \quad 20s < kT < 30s \end{cases} \quad (49)$$

where $T = 0.1$. In this simulation, we set $x_0 = [1 \quad 1]^T, \hat{x}_{0|0} = [0.5 \quad 1.5]^T$ and $P_{0|0} = 0.5 I_{2\times2}$.

To evaluate the performance of the proposed filters, two cases are considered:

A) It is assumed that the input u_{k-1} is unknown, so zero is replaced with u_{k-1} in (47).

B) It is assumed that the input u_{k-1} is known.

The parameters in the robust CKF algorithm are selected as $B = 0.2I_{2\times2}$ and $A = 0.8I_{2\times2}$. The initial mean of uncertainty is assumed to be $\widehat{w}_0 = [0 \quad 0]^T$ and initial uncertainty covariance is $P_{w,0} = Q$. The scaling parameter and the moving window width are selected as $\gamma = 1.5$ and $s = 4$ respectively, for the hybrid robust CKF. Performance of the three filters has been compared according to the RMSE of $x_{2,k}$ for 60 Monte Carlo simulation runs. Simulation results show that when the input is unknown, the robust CKF and hybrid robust CKF give better results in comparison with the CKF (fig 7). Furthermore, when the input is known, the CKF and hybrid robust CKF give better results in comparison with the robust CKF (fig 8). So, as it can be seen in the figures, in both cases, the hybrid robust CKF provides the best performance among all of the filters.

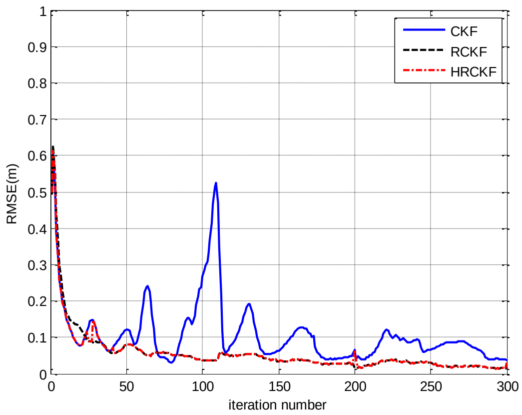

Fig. 7. RMS estimation error of x_2 with unknown input.

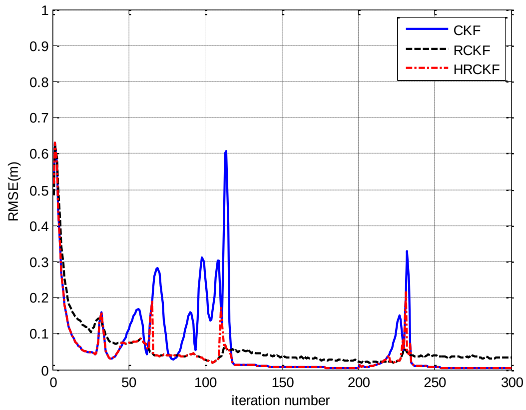

Fig. 8. RMS estimation error of x_2 with known input.

7. Conclusion

In the presented work, two novel methods of state estimation in nonlinear systems were proposed named robust CKF and hybrid robust CKF. The robust CKF was designed by including the uncertainty estimator in the traditional CKF which produces reliable estimates in presence of large modelling errors. The hybrid robust UKF was proposed to maintain a balance between uncertainty and estimation error. The hybrid robust CKF detects the uncertainty and adapts the system accordingly. Two examples have been considered to compare the performances of the proposed filters and it was found that the hybrid robust CKF provides the best results for any nonlinear system in the presence or absence of uncertainty.

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