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Nonlinear Regression Model Based on Fractional Bee Colony Algorithm for Loan Time Series

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Abstract

High levels of nonperforming loans provide negative impacts on the growth rate of gross domestic product. Therefore, predicting the occurrence of nonperforming loans is a vital issue for the financial sector and governments. In this paper, an intelligent nonlinear model is proposed for describing the behavior of nonperforming loans. In order to find the optimal parameters of the model, a new fractional bee colony algorithm (BCA) based on fractional calculus techniques is proposed. The inputs of the nonlinear model are the loan type, approved amount, refund amount, and economic realm. The output of the regression model is that whether the current information is for a nonperforming loan or not. Consequently, the model is modified to detect the status of a loan. So, the modified model predicts the occurrence of a nonperforming loan and determines the loan status, i.e., current, overdue, and nonperforming. The proposed procedure is applied to data gathered from an economic institution in Iran. The findings of this study are helpful for the managers of banks, and financial sectors to forecast the future of the loans and, therefore, manage the budget for the upcoming loan requests.

Keywords: Artificial Bee Colony; Fractional Calculus; Nonlinear Economic Model; Loan Status Prediction.

1- Introduction

1-1- Motivation

Banks and financial institutions have played a significant role in balancing the financial status of the people in recent decades owing to the development of the countries and the development of new financing opportunities for the merchants. Lending is one of the primary and popular approaches in financial organizations aiming to loan somebody if the amount borrowed is returned, usually with an interest fee. However, in some cases, the problem of nonperforming loans (NPL) occurs when the borrowed money is not returned in the scheduled period. High levels of NPL mean reducing the income of the banks, which in turn leads to their severe economic losses. Therefore, governments have paid more attention to this issue in recent years. Accordingly, a weakening in bank loan services may cause a delay in economic growth and can be a good reason for the economic crisis. It has been argued that NPLs may create economic stagnation and, therefore, deter economic growth and weaken financial efficacy [1]. On the other hand, high levels of nonperforming loans harm the growth rate of gross domestic product [2]. Given the importance of this issue, the governments and financial sectors should encourage researchers to conduct studies on this issue and banks to implement research results in practice.

Most of the studies have attempted to test whether a particular financial or banking attribute influences NPLs quotient or not. In most cases, just some measurable quantities which depend on the underlying and usually unknown dynamics of the NPLs rate are available. Since many factors are affecting the ratio of NPLs, the inherent nature of NPLs becomes more complicated and nondeterministic. In such circumstances, linear estimators that minimize the variance fail to achieve forecasting objectives. Therefore, it is hard to predict and interpret the long-term future of NPLs with limited features. The complex behavior of the NLPs cannot be easily modeled using the common linear or nonlinear statistical approaches such as autoregressive methods. Moreover, in the previous nonlinear neural network models, the designer should set many model parameters correctly. Thus, it would be better to use alternative nonlinear

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powerful techniques that utilize the NPLs nature's inherent attributes.

1-2- Contributions

With over 300 years of history, Fractional calculus is a generalization of ordinary differentiation and integration to arbitrary non-integer order. Although it has a long history, it has been used in physics and engineering during the past three decades. It has been recognized that many systems in interdisciplinary fields can be elegantly described using fractional-order differential equations. Moreover, fractional calculus has some exciting features, such as memory of all past events and the precise modeling ability of real-world systems. Using the powerful features of fractional calculus can improve the efficiency of the heuristic algorithms such as BCA.

This paper aims to propose a nonlinear polynomial regression model to forecast the NPLs of the main interest-free institution of Iran named Omid Entrepreneurship Fund (OEF). In order to find the optimal parameters of the proposed nonlinear regression model, a novel fractional heuristic method based on BCA is proposed to achieve a high-speed convergence rate with a high ability to escape local optimum traps. Based on the literature review, fractional BCA was not utilized for loan time series modeling to the best of our knowledge. In simulations, the proposed model is implemented for the data gathered from OEF. Numerical simulations show that this model can predict the future of a loan better than the original BCA and multi-layer perceptron neural network (MLPNN).
Contributions of this paper are as follows.

- Introducing nonlinear fractional BCA (FBCA) as a nonlinear polynomial regression model
- Constructing the four-input single-output heuristic model
- Considering loan type (LT), approved amount (AA), refund amount (RA), and economic filed (EF) as inputs of regression model
- Introducing two regression models for loan status prediction
- The output of the first model is whether the current information is for a nonperforming loan or not
- The second proposed intelligent regression model is modified to detect the exact status of a loan as current, overdue, and nonperforming

This paper is organized as follows. In Section II, the nonlinear regression model is presented. Section III deals with the description of the proposed FBCA. In Section IV, some computer simulations are carried out. Finally, concluding remarks are provided in Section V.

2- Related Works

NPL ratios should stand at low or manageable levels before the crisis. To this end, the NPL ratio prediction method based on previous information is necessary. Various studies have attempted to study the relationship between various economic factors and the NPLs to forecast the NPLs. In [1], a heuristic hybrid classification method has been used to predict the banks' nonperforming loans using some macroeconomic and bank-specific features. In [3], a step-wise discrimination algorithm was introduced to find essential factors for building distance discrimination and Bayesian discrimination models to determine whether an NPL has a zero or positive recovery rate. Three-phase mixture models of logistic regression and artificial neural networks have been developed in [4] to create an economic distress warning system appropriate for Taiwan's banking business. The application of a neural network predictor in forecasting loan recovery in Nigerian financial institutions has been reported [5]. In [6], the principal component analysis technique has been adopted for feature selection of Chinese bank loan default prediction models; then, the models have been assessed using the technique for order preference by similarity to ideal solution (TOPSIS). Authors in [7] have utilized machine learning strategies such as ontology, text and data mining, and multi-agent approaches to develop knowledge-determined automatic acquiescence auditing methods for bank loans. Some studies have used other techniques such as nonlinear regression models for loan prediction [8, 9]. In [10], Novikov's theorem has been utilized to model complicated dynamics of noisy credit risk contagion with time-delay, and the Hopf bifurcation and chaotic behaviors have been evaluated. In [11], some numerical approaches have been applied to discover Hopf bifurcation, inverse bifurcation, and chaos phenomena in the credit risk contagion dynamics. Lahmiri [12] has investigated the fractal inherence and chaotic behavior in returns and volatilities of family business companies of Morocco, using Hurst exponent and an autoregressive model. In [13], the phase synchronization method has been introduced for analyzing the chaotic behavior of stock price and index movements in crisis stages. For identifying the quality of similarity measure of financial time series, three techniques, including information categorization approach, reconstructed phase space clustering strategy, and system methodology with squared Euclidean distances, have been used in [14]. In addition to these works, some studies are aiming at predicting the chaotic financial time series. In [15], a self-organizing map (SOM) neural network and a recommender system have been proposed to cluster and predict stock price time series. Authors in [16] have applied empirical mode decomposition (EMD) and phase spacer construction methods combined with extreme learning machines (ELM) for predicting financial exchange rates' time series forecasting. In [17], the recovery rate of NPL for a European country was modeled using several linear regression models, including linear, linear with Lasso, beta, and inflated beta, as well as a two-stage beta mixture model combined with a logistic regression model. These models were used to estimate future recovery rates for improved risk assessment, capital requirement calculations, and bad debt management. The results showed that the two-stage model outperforms the others. A two-stage model including classification tree-based boosting and support vector regression (SVR) was proposed in [18] in order to estimate the loss given default (LGD). The results indicate that incorporating nonlinearity and boosting improve performance. Artificial intelligence is the primary and most applied technique in the literature for predicting the future of the time series [19-23].

Authors in [24] presented an ensemble-based machine learning model for predicting agricultural loans. For the building model, the dataset was gathered from an agricultural bank in Egypt. Variable selections were used to select important features for the classification. Some classifiers such as logistics regression, k-nearest neighbors, support vector machine, decision tree, and Meta-classifier methods were utilized for training and testing the dataset. The empirical impact of deprived sector lending on the nonperforming loans of commercial banks in Nepal was investigated in [25] that utilizes the ordinary least squares regression. The study establishes empirical relation between deprived sector lending and nonperforming loan of banks. A systematic literature review approach and discussion on the ten-year evolution of credit risk research was performed in [26]. It was found that machine learning is being extensively applied in credit risk assessment, where artificial intelligence applications mainly were found, more specifically artificial neural networks. The study of [27] compares Bayesian networks with artificial neural networks for predicting a recovered value in a credit operation. It was found that ANNs are a more efficient tool for predicting credit risk than the naive Bayesian approach. The convolutional neural network was employed in [28] to automatically extract essential cross features and generate cross-feature embedding from structured data, reducing the need to generate hand-crafted cross features. The local logit regression was employed in [29] for defaulted loan recoveries, and it was found that this model is robust to nonlinearity and non-normality of errors. Also, the empirical features of the local logit model were exploited to improve the specification of the standard regression for the fractional response variable model. The credit defaulter dataset of Bangladeshi banks was considered in [30], and several traditional machine learning classifiers were utilized to predict the delinquent clients who possessed the highest probability of short-term credit recovery. The different models for predicting the recovery rate on borrower level, including linear and quantile regressions, decision trees, neural networks, and mixture regression models, were compared in [31]. Authors in [32] proposed a system for building accurate models using interpretable state-of-the-art ML algorithms and explainable artificial intelligence techniques to explain individual instances for supporting business decisions. The basic user information and added the user's consumption features were considered in [33] to construct a loan risk assessment model that integrates features. Consumption features were extracted from two aspects: portrait features and sequence features. Finally, the features are combined and fed into the fully connected layer, and the probability of default loan is calculated.

Heuristic algorithms are higher-level procedures that are designed to find an excellent solution to optimization problems. Over the last decades, many intelligent algorithms have been developed to solve engineering optimization problems and time series forecasting [34-37]. Most of the developed algorithms are based on linear or nonlinear programming approaches. However, some complex problems cannot be solved with linear or nonlinear programming ways. For example, if the problem has more than one locally optimal solution, these methods must start with different initial points. Heuristic alternatives can find optimal solutions in complex problems using their capabilities (combination of randomness and rules, high speed, etc.). Bee colony algorithm (BCA) is one of the heuristic optimization methods introduced in 2005 by Dervis Karaboga [38]. The intelligent behavior of honey bees inspires the basic idea of BCA. Such as other swarm optimization techniques, BCA has a population-based search strategy in which bees fly around in a multidimensional search space, and the employed and onlooker bees choose food sources depending on the experience of themselves and their nestmates adjust their positions. Similar to other numerical optimization algorithms, BCA faces some challenges. For example, BCA shows a slow convergence speed during the search process.

Moreover, BCA quickly falls into local minima when handling complex multimodal problems [39]. Therefore, some mechanisms are required to improve the quality of the original BCA. In order to improve the accuracy and efficacy of the BCA, this paper proposes an innovative fractional calculus-based method.

3- Fractional Calculus and Nonlinear Regression Model

In this section, first, some preliminaries for the fractional calculus are provided. Then, the proposed nonlinear regression model is introduced.

3-1- Fractional Calculus

Definition 1. The fractional integration of order of function $f(t)$ in the sense of Riemann-Liouville is as follows [40]:

$$ {}_{t_0}I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau \qquad (1) $$

where t_0 is the initial time and the Euler's Gamma function $\Gamma(\alpha)$ is defined as:

$$ \Gamma(\alpha) = \int_0^{\infty} z^{\alpha-1} e^{-z} dz \qquad (2) $$

Definition 2. The Riemann-Liouville fractional derivative of the order $\alpha \in$ ¡ of function $f(t)$ is given by [40]:

$$ \begin{aligned} {}_{t_0}^{RL}D_t^{\alpha} f(t) &= \frac{d^{\alpha} f(t)}{dt^{\alpha}} \\ &= \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau \end{aligned} \qquad (3) $$

where $m-1 < \alpha \le m,\ m \in N$.

Definition 3. Grunwald–Letnikov fractional derivative is defined as [40]:

$$ {}_{t_0}^{GL}D_t^{\alpha} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\left[\frac{t-t_0}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh) \qquad (4) $$

where $[.]$ is the integer part operator and $\binom{\alpha}{j}$ is the fractional binomial coefficient defined by:

$$ \binom{\alpha}{j} = \begin{cases} 1 & j = 0 \\ \dfrac{\alpha(\alpha-1)...(\alpha-j+1)}{j!} & j > 0 \end{cases} \qquad (5) $$

Remark 1. Eq. (3) reveals that although the integer-order operators are represented by finite series, the fractional-order counterparts are given via infinite series. This indicates that integer-order derivatives are local operators in contrast to the fractional operators that have, implicitly, a memory of all past events. So, it can be concluded that the fractional-order systems have limitless memory (infinite-dimensional) while the integer-order systems posse limited memory (they are finite-dimensional).

Remark 2. A discretization scheme using a finite difference equation should apply fractional-order derivatives and integrals in real applications. The well-known and most straightforward technique is to benefit memory length expansion from (4) to construct a direct discretization. This methodology relies on Grunwald-Letnikov, and Riemann-Liouville's definitions are equivalent in reasonable conditions for a broad class of functions [40]. Accordingly, the following definition of the approximated fractional difference is adopted in this article:

$$ \Delta_h^{\alpha} x(t) = \frac{1}{h^{\alpha}} \sum_{j=0}^{k} (-1)^j \binom{\alpha}{j} x(t-jh) \qquad (6) $$

where $k \in$ ¥ shows the number of samples for which the approximation of the derivative is computed.

3-2- Nonlinear Regression Model

In this paper, we propose a nonlinear regression model (NRM) based on FBCA to predict the status of the loans. The nonlinear regression attempts to find the functional relationship between the inputs and outputs. Here, the coefficients of the nonlinear regression are estimated by the FBCA. The proposed FBCA is designed to minimize the mean square error (MSE) between the predicted and target outputs. In this paper, the following nonlinear polynomial regression model is used:

$$ F_t = K + \sum_{i=1}^{n} k_i x_i + \sum_{i=1}^{n} q_i x_i^{r_i} + \sum_{i=1}^{n} \sum_{j=i+1}^{n-1} p_{i,j} x_i x_j \qquad (7) $$

where K , k_i , q_i , r_i , and p_{ij} are constant regression parameters and F denotes the loan status.

Remark 3. It should be noted that FBCA determines the fair values for the constant parameters of the model.

Remark 4. The proposed regression model involves two phases; train and test. The training data is applied to the model in the training phase, and the output error is computed. The model parameters are computed using FBCA to minimize this error. After *achieving a less enough train error*, the model parameters are fixed. The test phase is performed on unseen data to evaluate the model's generalization ability for the new data.

Remark 5. The mean square error (MSE) is calculated as:

$$ MSE = \frac{\sum_{t=1}^{N} (F_t - Y_t)^2}{N} \qquad (8) $$

where Y_t represents the desired output and N denotes the number of data.

4- The Proposed FBCA Method

In this section, first, the original BCA is restated. Then, the proposed FBCA is introduced.

4-1- The Original BCA

As an intelligent optimization tool, BCA provides a population-based search scheme such that the artificial bees modify individuals' called foods positions with time.

In this process, the bee's goal is to look for food sources with high nectar amounts and, at last, the one with the highest nectar. In the ABC system, artificial bees fly around in a multidimensional search space, and some (employed and onlooker bees) select food sources based on their experience and their nestmates and regulate the corresponding positions. Some (scouts) fly and select the food sources in a stochastic manner with no experience. The bees will memorize the new position and forget the previous one if the nectar amount of a new source is higher than that in their memory. Hence, the BCA system combines local search techniques, performed by employed bees and onlooker ones, with global search approaches modeled by onlookers and scouts. This process tries to balance exploration and exploitation.

The first half of the population comprises employed bees in the BCA scheme, and the second half composes the onlooker bees. The number of employed bees or the onlooker bees is set as the number of solutions in the population [26].

The BCA produces a stochastically distributed initial swarm of N_{pop} solutions. Assume $\mathbf{x}_i = \{x_{i1}, x_{i2}, ..., x_{id}\}$ represents the ith solution in the swarm, where d is the dimension size. Each employed bee $\mathbf{x}_i$ produces a new potential solution v_i in the neighborhood of its current position as:

$$v_{ij} = x_{ij} + \varphi_{ij}(x_{ij} - x_{kj}) \tag{9}$$

where x_{kj} is a randomly selected candidate solution $(k \neq i)$, j stands for a random dimension index selected from the set $\{1, 2, ..., d\}$, and φ_{ij} represents a random number within [-1, 1]. Once the new candidate solution v_i is built, a greedy selection is adopted. If the fitness value v_i is higher than that of its parent v_i, then update $\mathbf{x}_i$ with v_i; otherwise $\mathbf{x}_i$ is held unchangeable.

After all employed bees complete the search process, they share the information about their food sources with the onlooker bees through waggle dances. An onlooker bee assesses the nectar information adopted from all employed bees and chooses a food source with a probability related to its nectar amount. This probabilistic selection is equal to a roulette wheel selection process which is given as:

$$p_i = \frac{\gamma_i}{\sum_{j=1}^{N_{pop}} \gamma_j} \tag{10}$$

where γ_i is the fitness value of the ith solution in the population. It is clear that the better ith solution, the higher the probability of the ith food source being chosen. The food source is abandoned if a position cannot be improved over a predefined number of cycles. Letting the abandoned source as $\mathbf{x}_i$, the scout bee finds a new food source to be exchanged with $\mathbf{x}_i$ as:

$$x_{ij} = b_j + \beta(u_j - b_j) \tag{11}$$

where β is a random number uniformly distributed within the range [0, 1] and b_j and u_j indicate the lower and upper boundaries of the jth dimension, respectively.

4-2- The FBCA Algorithm

Globally optimizing a fitness function in a given search domain finds its global optima fast and without sticking in local optima. Slow convergence of BCA before achieving an exact solution is a drawback related to its lack of appropriate memory in the bee position update (9). One can rewrite (9) as follows:

$$v_{ij} - x_{ij} = \varphi_{ij}(x_{ij} - x_{kj}) \tag{12}$$

It is clear from $v_{ij} = x_{ij}(t+1)$, where t stands for the time or iteration, and the left side of (12) $v_{ij} - x_{ij}$ is the discrete version of the fractional derivative. In other words, the left side of (12) represents the fractional difference (6) with order $\alpha = 1$ and $h = 1$, i.e.,

$$\Delta_1^1 v_{ij} = \varphi_{ij}(x_{ij} - x_{kj}) \tag{13}$$

Also, using a real order difference (i.e., fractional difference) can result in a more generalized position updating relation. So, generalization of the fractional difference order to a real number $0 \le \alpha \le 1$ may access a smoother variation and a longer memory effect. Thus, considering the first $k = 4$ terms of the fractional difference given by (6), the following fractional position updating relation is proposed:

$$\begin{aligned} &v_{ij} - \alpha x_{ij}(t) - \frac{1}{2}\alpha x_{ij}(t-1) - \frac{1}{6}\alpha(1-\alpha)x_{ij}(t-2) \\ &-\frac{1}{24}\alpha(1-\alpha)(2-\alpha)x_{ij}(t-3) = \varphi_{ij}(x_{ij} - x_{kj}) \end{aligned} \tag{14}$$

or

$$\begin{aligned} &v_{ij} = \alpha x_{ij}(t) + \frac{1}{2}\alpha x_{ij}(t-1) + \frac{1}{6}\alpha(1-\alpha)x_{ij}(t-2) \\ &+\frac{1}{24}\alpha(1-\alpha)(2-\alpha)x_{ij}(t-3) + \varphi_{ij}(x_{ij} - x_{kj}) \end{aligned} \tag{15}$$

4-3- Loan Status Prediction by FBCA-Based Nonlinear Regression

In this study, the nonlinear regression based on modified BCA is used to forecast the loans' future status. As mentioned before, nonlinear regression is a regression analysis that uses a combination of the independent variables to solve the nonlinear problems. Most of the studies have been used optimization algorithms and neural networks to achieve the best weights for the NRM. In this study, the introduced FBCA search is used to estimate the best weights for the NRM. Some essential features of a loan are considered as the inputs of the regression model to make a prediction. In this study, the inputs are selected as the features loan type (LT), approved amount (AA), refund amount (RA), and economic filed (EF), which are available from the data obtained from OEF. LT and EF are qualitative variables, while AA and RA are quantitative variables. The inputs LT, AA, and RA are the loan features, while EF is not dependent on the loan and is related to the scope of application; hence, the proposed loan status prediction can be considered multimodal.

Two structures are chosen for the output of the model. In the first structure, we are attempting to detect that whether a loan with a known feature vector is related to an NPL or not. In this case, the model has a single output. The second structure is proposed for classification (and therefore for prediction) of the loans' status, including current loan (CL), overdue loan (OL), and NPL. In this case, the model has several outputs. In both cases, the model estimates the loan status only based on its corresponding input vector. However, the main issue is selecting optimal parameters (weights) for the model. Based upon previous discussions, the proposed FBCA is applied to find the optimal weights while minimizing the MSE (or maximizing inverse of MSE). After finding the suitable weights, the model structure is fixed, and the model's generalization capability is examined in the test mode using unseen data. The main procedure of the proposed scheme for loan status prediction is given by the pseudo-codes provided in Algorithms 1 and 2.

5- Data and Numerical Analysis

In this section, first, a brief description of the adopted data is given. Then, the efficiency of the proposed FBCA is tested using a set of benchmark functions. Also, comprehensive numerical simulation and analysis are provided to verify the data's complex behavior and forecast the future status of the loans.

5-1- Financial Data

Recently, some microfinance institutes have the mission of providing limited loans with no (or at least a minimum) interest. The target population of such interest-free organizations is low-income people who lack access to the financial services of other banks or traditional financial institutions. The essential condition for requesting an interest-free loan is that the recipient must prove setting up a small-scale enterprise. The essential goal of such microfinance institutes is to help low-income people get better access to financial services and finance small or medium projects. OEF is the most essential and essential interest-free institution of Iran. OEF has at least one branch in each province of Iran. The five-year loan features of two branches of OEF with the highest level of activity in the considered time horizon have been chosen and call them B_1 and B_2 branches. Since an NPL is defined as a loan with no payment for at least 18 months for the OEF policies, the last two years' data (which do not contain an NLP) are removed. The major statistical attributes for the numerical features of the original input data are shown in Table 1. It is noted that in all simulations, all data is normalized to the range [-1, 1].

Algorithm 1: Train phase of the proposed FBCA

Step 1. Choose the inputs (loan features) and outputs (loan status) of the model.

Step 2. Initialize the parameters of the model (such as maximum iteration, population size, etc.).

Step 3. Set the weights of the regression model as the decision variables.

Step 4. Randomly initialize the swarm.

Step 5. For each employed bee, generate a new candidate solution v_i according to (15).

Step 6. Evaluate the fitness of v_i using the regression model (7) and the inverse of MSE (8).

Step 7. Use a roulette wheel selection (10) to choose a better one between $\mathbf{x}_i$ and v_i as the new $\mathbf{x}_i$.

Step 8. The scout bee determines the abandoned $\mathbf{x}_i$, if exists and update it by (11).

Step 9. Update the solution pool, and the best solution found so far, and iteration increases one unit.

Step 10. If the number of iterations reaches the maximum value, stop the algorithm and output the results; otherwise, go to Step 5.

End of train phase.

Algorithm 2: The test phase of the proposed FBCA

Step 1. Fix the parameters (weights) of the model (7) using the best solution found in the Train phase.

Step 2. Apply the test data to the fixed model and obtain the outputs.

Step 3. Compute the error (8).

End of the Test phase.

Table 1: Statistical attributes of the input data

Branch	B_1		B_2	
Number of data	1661		1531	
Input feature	AA	RA	AA	RA

Mean	8.40×10^{7}	68.43	1.42×10^{8}	54.67
Median	9.0×10^{7}	80.7800	1.0×10^{8}	50.01
Min.	5.0×10^{6}	0	1.0×10^{7}	0
Max.	1.0×10^{9}	100	4.0×10^{10}	100
Standard deviation	6.72×10^{7}	34.59	1.02×10^{9}	36.11
Variance	4.52×10^{15}	1.1965×10^{3}	1.05×10^{18}	1.30×10^{3}

5-2- Testing Efficiency of FBCA

Here, the efficiency of the proposed FBCA is verified using a set of benchmark functions. To avoid any misinterpretation of the optimization results related to the choice of any particular initial parameters, we performed each test 50 times, starting from various randomly selected solutions drawn from the standard uniform distribution on the search domain specified in the usual literature. The results of FBCA tests on eight functions given in Table 2 are provided in Table 3. To assess the effectiveness of the proposed FBCA procedure, the rate of successful minimizations (r_{SM}), the average of the objective function evaluation numbers (n_{obj}), and the average error $(\bar{\varepsilon})$ are taken into account as minimization criteria. The rate of successful minimizations denotes the ratio of the number of trials that FBCA reaches the minimum value to the total number of evaluation trials. Hence, it can be formulated as: $r_{SM} = n_{\min} / n_{total}$, where $n_{\min}$ is the number of trials FBCA reaches the minimum value and n_{total} is the total number of evaluation trials. The objective function evaluation number denotes the number of iterations of FBCA before reaching the objective function to the minimum value or the number of iterations performed before the convergence of FBCA.

Table 3: Results of FBCA for eight benchmark functions

Function	r_{SM} (%)	n_{obj}	$\bar{\varepsilon}$
SH	100	43	0.000
ES	100	39	0.000
S_n	100	86	0.000
NQ_n	99	110	0.001
RA_n	99	115	0.001
SC_n	99	112	0.001
R_n	99	138	0.002
Z_n	99	142	0.002

The performance of FBCA is then compared to the original BCA and GA algorithms. The experimental results obtained for the test functions, using the three different methods, are given in Table 4. In our simulations, each population in GA has 15 chromosomes, and a swarm in BCA has 15 particles. Other parameters of the three algorithms are selected by trial and error. For each function, we give the average number of function evaluations for 100 runs.

Table 4: The average number of objective function evaluations

Function	FBCA	BCA	GA
SH	43	132	224
ES	39	128	201
S_n	86	164	416
NQ_n	110	237	513
RA_n	115	389	650
SC_n	112	411	780
R_n	138	583	869
Z_n	142	691	979

The convergence plots of the three methods for two functions Sn and NQn, are illustrated in Figs. 1-2. One sees that in Fig. 1, the GA is trapped in a local minimum, and the original BCA spends more time finding the global minima.

Table 2: The benchmark test functions

Name	Function	D	Search space	$\mathbf{x}^*$	$f\left(\mathbf{x}^*\right)$
SH	$f_1(\mathbf{x}) = \left(\sum_{j=1}^{5} j\cos((j+1)x_1+j)\right)\left(\sum_{j=1}^{5} j\cos((j+1)x_2+j)\right)$	2	$[-10,10]^D$	18 minimum	-186.7
ES	$f_2(\mathbf{x}) = -\cos(x_1)\cos(x_2)\exp(-((x_1-\pi)^2+(x_2-\pi)^2))$	2	$[-100,100]^D$	$[-\pi, \pi]$	-1
S_n	$f_3(\mathbf{x}) = \sum_{i=1}^{D} x_i^2$	10	$[-100,100]^D$	$[0,0,...,0]$	0
NQ_n	$f_4(\mathbf{x}) = \sum_{i=1}^{D} i x_i^4$	15	$[-10,10]^D$	$[0,0,...,0]$	0
RA_n	$f_5(\mathbf{x}) = \sum_{i=1}^{D}\left(x_i^2 - 10\cos(2\pi x_i)+10\right)$	20	$[-5,5]^D$	$[0,0,...,0]$	0
SC_n	$f_6(\mathbf{x}) = \sum_{i=1}^{D} -x_i \sin\left(\sqrt{\lvert x_i \rvert}\right)$	20	$[-500,500]^D$	$[1,1,...,1]$	$-418.98D$
R_n	$f_7(\mathbf{x}) = -\sum_{i=1}^{D} -\left(100(x_i^2 - x_{i+1})^2 + (x_i-1)^2\right)$	25	$[-5,10]^D$	$[1,1,...,1]$	0
Z_n	$f_8(\mathbf{x}) = \sum_{i=1}^{D} x_i^2 + \left(\sum_{i=1}^{D} 0.5ix_i\right)^2 + \left(\sum_{i=1}^{D} 0.5ix_i\right)^4$	30	$[-5,10]^D$	$[1,1,...,1]$	0

Also, from Fig. 2, both GA and BCA cannot find the optimum global point. However, the proposed FBCA can achieve the global minimum solution with fewer function evaluation numbers. Therefore, it can be concluded that the FBCA gains more accurate results compared to the original BCA and GA methods. Moreover, FBCA is faster than the GA and BCA methods.

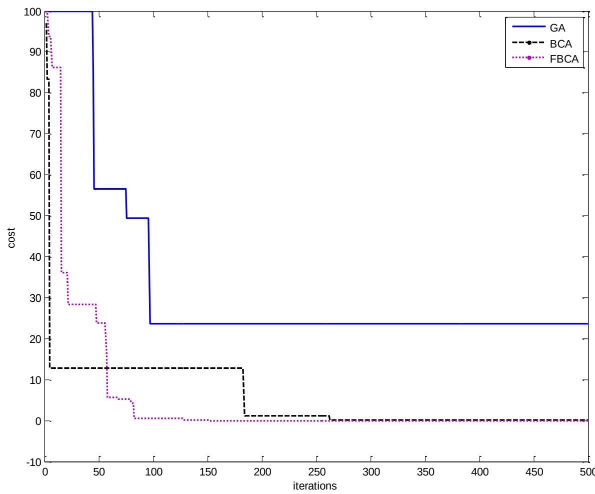

Fig. 1 Typical convergence diagrams of the three methods for S_n function

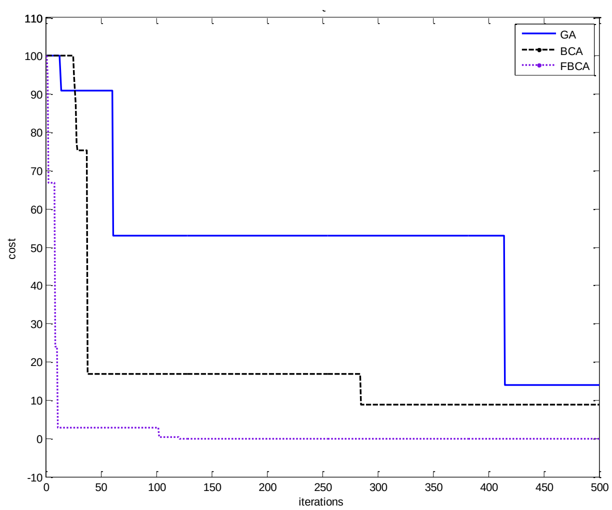

Fig. 2 Typical convergence diagrams of the three methods for NQ_n function

5-3- NPL Prediction

We attempted to forecast the occurrence of NLPs with the heuristic nonlinear prediction method. In this case, a nonlinear regression model was developed using the proposed FBCA. Based on simulations, we assume that if the model's output (7) is in the range (0,100], it denotes an NLP, and if the output is within (100, 200], it does not stand for an NPL. Outranges are missed, and the simulation is repeated for them. Based on the mentioned pseudo-code, the parameters of the model (7) are found. The initial parameters of the optimization algorithm are chosen as follows: $n = 50$, maximum number of iterations = 1000, $p = 0.1$ and $\alpha = 0.05$. To compare the results, a feed-forward MLPNN with one hidden layer of 10 neurons is also implemented to predict the output of the loans. To evaluate the performance of the algorithms, Precision, Recall, and F-measure criteria are adopted as follows:

$$\text{Precision}=\frac{TP}{TP+FP} \tag{16}$$

where TP denotes the number of correctly detected loans as NPLs and FP is incorrectly identified as NPLs. The recall is defined as the ratio of the number of correct loan classes to the sum of the corrected ones with the loans which are misidentified:

$$\text{Recall}=\frac{TP}{TP+FN} \tag{17}$$

where FN is the number of loans that are incorrectly labeled as NLPs. F-measure is also defined as a measure of the test accuracy:

$$\text{F-measure}=2\frac{\text{Precision}\times\text{Recall}}{\text{Precision}+\text{Recall}} \tag{18}$$

Table 5 compares the results of both methods. It is observed that the proposed nonlinear intelligent model can detect the NLPs with high accuracy compared to the MLPNN. As a result, the predicted values can identify and predict NPLs before they become uncontrollable. Therefore, these predictions can help the managers of financial sectors to develop the necessary policies regarding the budget and the remainder of the loans.

Table 5: Comparative results for NPL detection using the proposed method and MLPNN

Method	**Precision**		**Recall**	
	B_1	B_2	B_1	B_2
Proposed method	0.9712	0.9800	0.9502	0.9471
BCA	0.9031	0.9082	0.9279	0.9206
MLPNN	0.8523	0.8491	0.9042	0.9154
Method	**F-measure**		**Number of function calls**	
	B_1	B_2	B_1	B_2
Proposed method	0.9606	0.9633	570	592
BCA	0.9153	0.9144	713	774
MLPNN	0.8775	0.8810	851	886

5-4- Status Prediction

There are three outputs for forecasting the status of the loans, including current, overdue, and nonperforming. Similar to the previous scenario, we assign a linear portion for each output. In this case, the first output gets the values in the range (0, 100], the second output gets (100, 200], the third output stands for (200, 300], and the final output is assigned to (300, 400]. Outranges are missed, and the simulation is repeated for them. The model's parameters (6) are calculated using the proposed procedure based on the mentioned pseudo-code. The initial parameters of the optimization algorithm are chosen as follows: n = 70, the maximum number of iteration = 1000, p = 0.1 and α = 0.05. To compare the results, a feed-forward MLPNN with one hidden layer of 20 neurons is also implemented to predict the output of the loans. To evaluate the performance of the algorithms, the last five criteria are considered as the comparison criteria. Table 6 shows the results of both techniques. The proposed nonlinear heuristic model outperforms the MLPNN in all criteria. Consequently, the proposed model can identify the status of the loans to help the managers of banks build up the necessary policies for the budget and the loans' customers. Although FBCA has more computational complexity than the BCA, it enhances the accuracy considerably.

Table 6: Comparative results for status prediction using the proposed method and MLPNN

Method	**Precision**		**Recall**	
	B_1	B_2	B_1	B_2
Proposed method	0.9742	0.9782	0.9520	0.9511
BCA	0.9061	0.9021	0.9183	0.9212
MLPNN	0.8751	0.8791	0.9172	0.9090
Method	**F-measure**		**Number of function calls**	
	B_1	B_2	B_1	B_2
Proposed method	0.9630	0.9645	602	627
BCA	0.9122	0.9115	751	808
MLPNN	0.8957	0.8938	884	922

The value of k denotes the number of fractional difference terms, and its value depends on the trade-off between complexity and accuracy. Considering the higher values for k increases both complexity and accuracy and vice-versa. Also, the value of α, which denotes the of fractional difference order, was chosen by trial and error. The lower values of α have smoother results, but reduce the generalization. Table 7 compares the performance of the proposed method for loan status prediction in terms of F-measure for several pairs of (α, k).

Table 7: Performance of the proposed method for different values of pair (α, k)

(α, k)	(0.01, 2)	(0.01, 4)	(0.01, 6)	(0.01, 8)
F-measure	0.9527	0.9719	0.9774	0.9793
(α, k)	(0.03, 2)	(0.03, 4)	(0.03, 6)	(0.03, 8)
F-measure	0.9508	0.9658	0.9689	0.9709
(α, k)	(0.05, 2)	(0.05, 4)	(0.05, 6)	(0.05, 8)
F-measure	0.9481	**0.9645**	0.9672	0.9694
(α, k)	(0.07, 2)	(0.07, 4)	(0.07, 6)	(0.07, 8)
F-measure	0.9472	0.9619	0.9651	0.9638

6- Concluding Remarks

This study provided a new nonlinear regression tool to detect the status of the loans using a limited knowledge about the features affecting the loans. To obtain an optimal model, a new version of BCA is constructed using fractional calculus theory. The effectiveness of the fractional optimization method is verified using test benchmark functions, and it is shown that the new algorithm is fast and reliable. We provide a flexible nonlinear heuristic model that can identify nonperforming loans and any status of the loans. The simulation results using data gathered to form a financial sector in Iran showed that the proposed scheme can effectively forecast the occurrence of NLPs. Moreover, the introduced approach can detect the status of the loan using an average input feature vector. The findings of this study are helpful for the managers of banks and financial sectors to use the nonlinear models to detect the potential nonperforming loans before they become uncontrollable.

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